

Dynamic Analysis

With Emphasis On

Wind and Earthquake Loads

BY

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Summary Of Presentation

1. General Comments
2. History Of The Development of SAP
3. Computer Hardware Developments
4. Methods For Linear and Nonlinear Analysis
5. Generation And Use Of **LDR** Vectors and Fast Nonlinear Analysis - **FNA** Method
6. Example Of Parallel Engineering Analysis of the Richmond - San Rafael Bridge

Structural Engineering Is

The Art Of Using Materials

Which We Do Not Fully Understand

To Build Structural Systems

Which Can Only Be Approximately Analyzed

To Withstand Forces

Which Are Not Accurately Known

So That We Can Satisfy

Our Responsibilities

In Regards To Public Safety

FUNDAMENTALS OF ANALYSIS

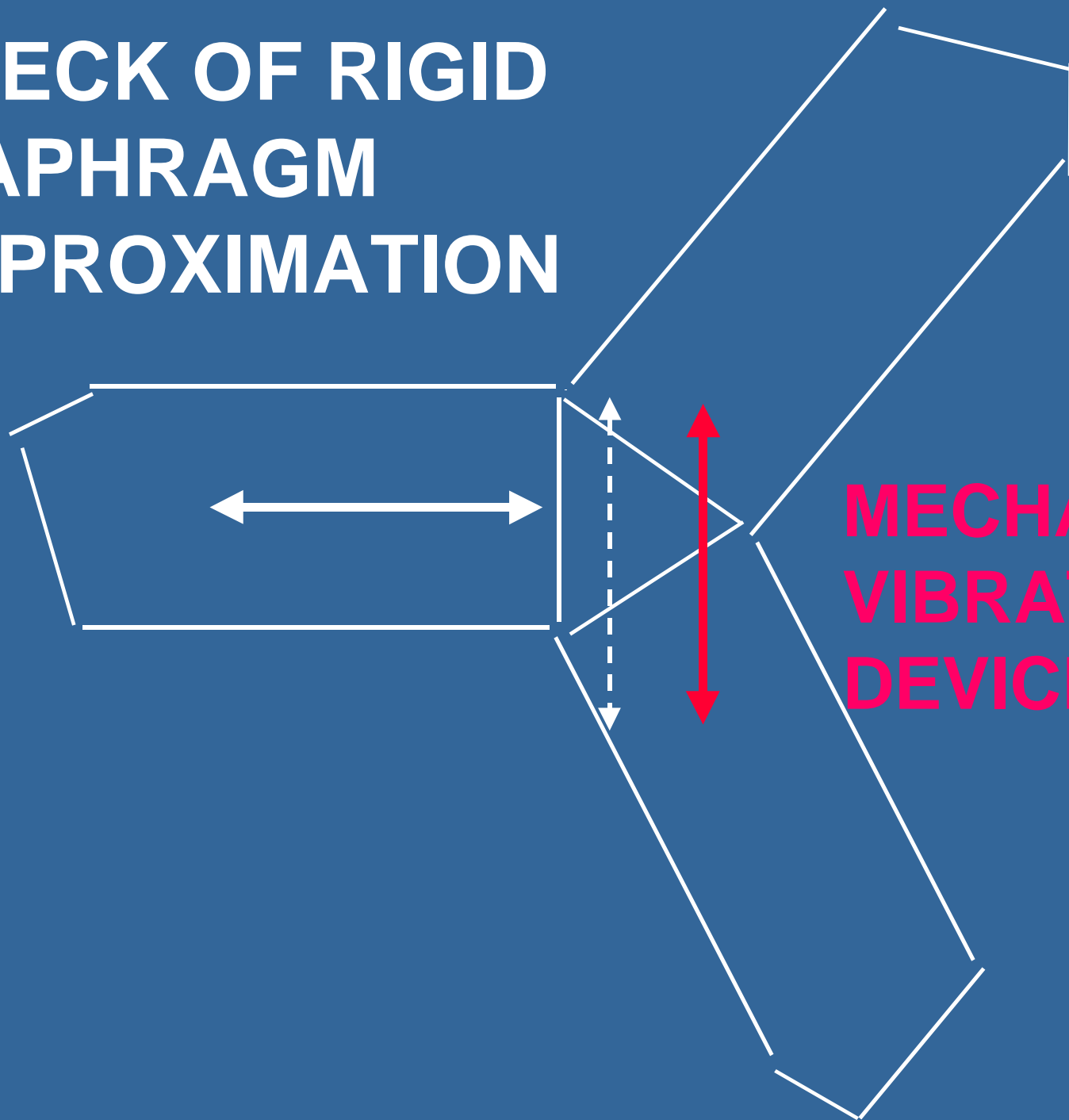
1. UNDERSTAND PHYSICS OF PROBLEM
2. CREATE COMPUTER MODEL
3. CONDUCT PARAMETER STUDIES
4. VERIFICATION OF RESULTS
STATIC AND DYNAMIC EQUILIBRIUM
ENERGY BALANCE
5. FIELD OR LABORATORY TESTS

FIELD MEASUREMENTS REQUIRED TO VERIFY

- 1. MODELING ASSUMPTIONS**
- 2. SOIL-STRUCTURE MODEL**
- 3. COMPUTER PROGRAM**
- 4. COMPUTER USER**



CHECK OF RIGID DIAPHRAGM APPROXIMATION



FIELD MEASUREMENTS OF PERIODS AND MODE SHAPES

MODE	T _{FIELD}	T _{ANALYSIS}	Diff. - %
1	1.77 Sec.	1.78 Sec.	0.5
2	1.69	1.68	0.6
3	1.68	1.68	0.0
4	0.60	0.61	0.9
5	0.60	0.61	0.9
6	0.59	0.59	0.8
7	0.32	0.32	0.2
-	-	-	-
11	0.23	0.32	2.3

FIRST DIAPHRAGM MODE SHAPE



15 th Period

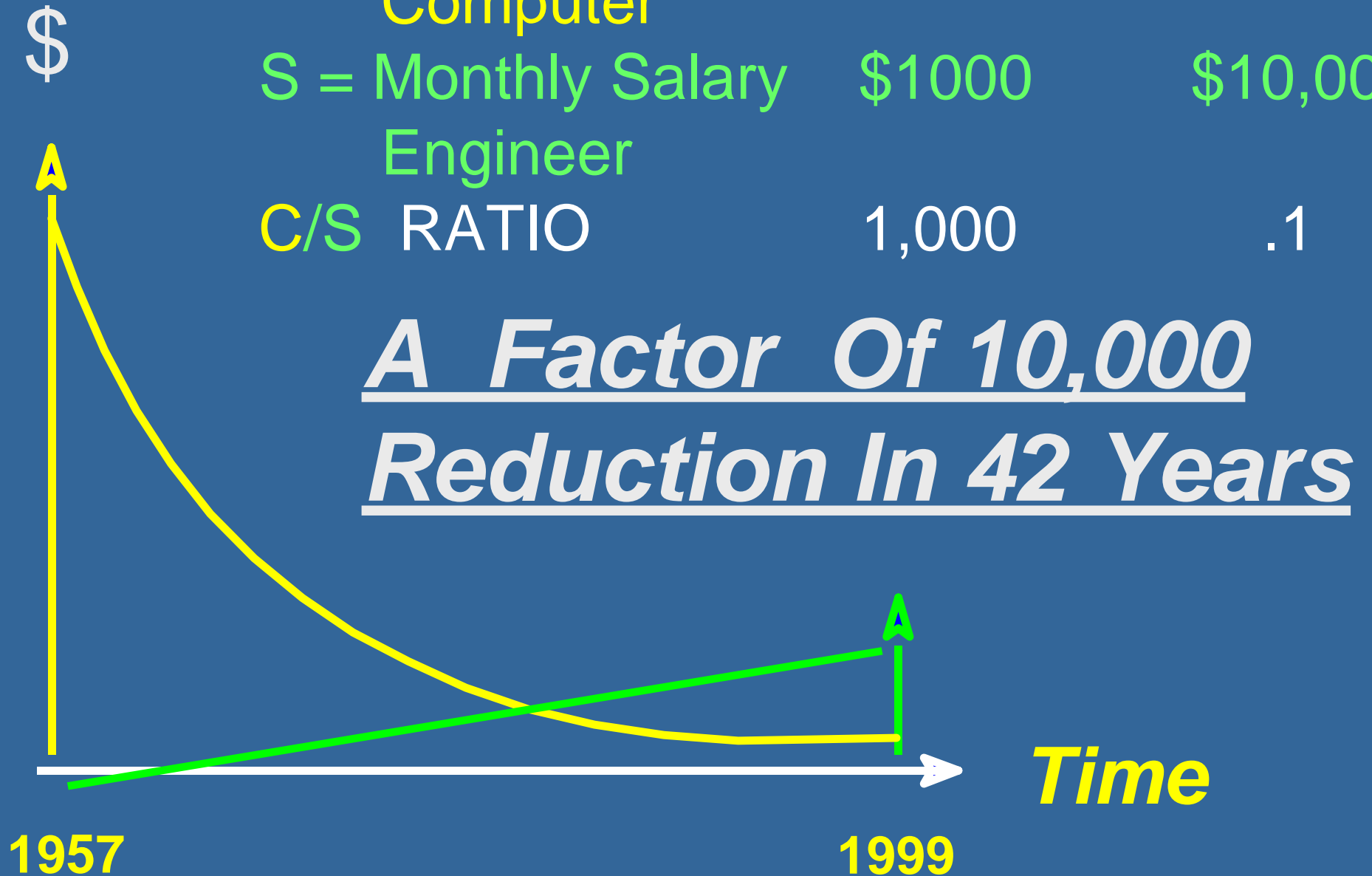
$T_{\text{FIELD}} = 0.16 \text{ Sec.}$

COMPUTERS

1957 TO 1999

IBM 701 - PENTIUM III

	<u>1957</u>	<u>1999</u>
C = Cost of Computer	\$1,000,000	\$1,000
S = Monthly Salary Engineer	\$1000	\$10,000
C/S RATIO	1,000	.1



Floating Point Speed Comparison

FORTRAN 64 bits - REAL*8

Definition of one Operation $A = B + C * D$

Year	COMPUTER	Op/Sec	Relative Speed
1963	CDC-6400	50,000	1
1967	CDC-6600	200,000	4
1974	CRAY - 1	3,000,000	60
1980	VAX - 780	100,000-	2-
1981	CRAY-XMP	30,000,000	600
1988	Intel 80387	100,000	2
1990	DEC-5000	3,500,000	70
1994	Pentium 90	3,500,000	70
1995	DEC - ?	14,500,000	280
1997	Pentium Pro	10,000,000	200
1998	Pentium II	17,000,000	350
1999	Pentium III	45,000,000	900

Floating Point Speed Comparison - PC

Microsoft FORTRAN 64 bits - REAL*8

Definition of one Operation $A = B + C * D$

Year	CPU	Speed MHz	Op/Sec	Normalized
1980	8080	4	200	1
1984	8087	10	13,000	65
1988	80387	20	93,000	465
1991	80486	33	605,000	3,025
1994	PENTIUM	66	1,210,000	6,050
1996	PENTIUM	133	5,200,000	26,000
1996	Pentium-Pro	200	10,000,000	50,000
1998	Pentium II	333	17,000,000	85,000
1999	Pentium III	450	45,000,000	225,000

The Sap Series

Structural Analysis Programs

1969 To 1999

S A P

**STRUCTURAL ANALYSIS
PROGRAM**

ALSO A PERSON

“ Who Is Easily Deceived Or Fooled ”

“ Who Unquestioningly Serves Another ”

From The Foreword Of The First SAP Manual

"The slang name S A P was selected to remind the user that this program, like all programs, lacks intelligence.

It is the responsibility of the engineer to idealize the structure correctly and assume responsibility for the results."

Ed Wilson 1970

The Sap Series Of Programs

1969	SAP	With User Defined Ritz Vectors
1971	SOLID SAP	For Static Loads Only
1972	SAP IV	With Full Dynamic Response
1973	NONSAP	Now ADINA
1980	SAP 80	NEW Program for PC , Elements and Methods
1983	SAP 80	CSI Added Pre and Design Post Processing
1989	SAP 90	Large Capacity on PC
1991	SADSAP	R & D Program With Nonlinear Elements
1997	SAP 2000	Added Graphical User Interface

S T A T I C

A N D

D Y N A M I C

S T R U C T U R A L

A N A L Y S I S

P R O G R A M

How Can Engineers Be Convinced To Use New And Improved Methods Of Analysis ?

1. Give Them New Capabilities Such As 2 and 3d Nonlinear Analyses
2. Or, The Program Must Be Easy To Use, Fast On A PC, And Have

FANCY COLORED GRAPHICS

SAP2000

A Good Computer Program

1. The Fundamental Equations Must Represent The Real Physical Behavior Of The Structure
2. Accurate , Efficient And Robust Numerical Methods Must Be Used
3. Must Be Programmed In Portable Language In Order To Justify Development Cost
4. Must Have User-friendly Pre And Post Processors
5. Ability To PLOT All Possible Dynamic Results As A Function of TIME - Only SAP 2000 Has This Option

***Numerical Methods for
The Seismic Analysis of
Linear and Nonlinear
Structural Systems***

DYNAMIC EQUILIBRIUM EQUATIONS

$$\mathbf{M} \mathbf{a} + \mathbf{C} \mathbf{v} + \mathbf{K} \mathbf{u} = \mathbf{F}(t)$$

\mathbf{a} = Node Accelerations

\mathbf{v} = Node Velocities

\mathbf{u} = Node Displacements

\mathbf{M} = Node Mass Matrix

\mathbf{C} = Damping Matrix

\mathbf{K} = Stiffness Matrix

$\mathbf{F}(t)$ = Time-Dependent Forces

PROBLEM TO BE SOLVED

$$M a + C v + K u = \sum f_i g(t)_i$$

$$= - M_x a_x - M_y a_y - M_z a_z$$

For 3D Earthquake Loading

**THE OBJECTIVE OF THE ANALYSIS
IS TO SOLVE FOR ACCURATE
DISPLACEMENTS and MEMBER FORCES**

METHODS OF DYNAMIC ANALYSIS

For Both Linear and Nonlinear Systems

STEP BY STEP INTEGRATION - $0, \Delta t, 2 \Delta t \dots N \Delta t$

USE OF MODE SUPERPOSITION WITH EIGEN OR
LOAD-DEPENDENT RITZ VECTORS FOR FNA

For Linear Systems Only

TRANSFORMATION TO THE FREQUENCY
DOMAIN and FFT METHODS

RESPONSE SPECTRUM METHOD - CQC - SRSS

STEP BY STEP SOLUTION METHOD

1. *Form Effective Stiffness Matrix*
2. *Solve Set Of Dynamic Equilibrium Equations For Displacements At Each Time Step*
3. *For Non Linear Problems Calculate Member Forces For Each Time Step and Iterate for Equilibrium - **Brute Force Method***

MODE SUPERPOSITION METHOD

- 1. Generate Orthogonal Dependent Vectors And Frequencies*
- 2. Form Uncoupled Modal Equations And Solve Using An Exact Method For Each Time Increment.*
- 3. Recover Node Displacements As a Function of Time*
- 4. Calculate Member Forces As a Function of Time*

GENERATION OF LOAD DEPENDENT RITZ VECTORS

- 1. Approximately Three Times Faster Than The Calculation Of Exact Eigenvectors*
- 2. Results In Improved Accuracy Using A Smaller Number Of **LDR** Vectors*
- 3. Computer Storage Requirements Reduced*
- 4. Can Be Used For Nonlinear Analysis To Capture Local Static Response*

STEP 1. INITIAL CALCULATION

A. TRIANGULARIZE STIFFNESS MATRIX

B. DUE TO A BLOCK OF STATIC LOAD VECTORS, \mathbf{f} ,
SOLVE FOR A BLOCK OF DISPLACEMENTS, \mathbf{u} ,

$$\mathbf{K} \mathbf{u} = \mathbf{f}$$

C. MAKE \mathbf{u} STIFFNESS AND MASS ORTHOGONAL TO
FORM FIRST BLOCK OF LDL VECTORS \mathbf{V}_1

$$\mathbf{V}_1^T \mathbf{M} \mathbf{V}_1 = \mathbf{I}$$

STEP 2. VECTOR GENERATION

$i = 2 \dots N$ Blocks

- A. Solve for Block of Vectors, $K X_i = M V_{i-1}$
- B. Make Vector Block, X_i , Stiffness and Mass Orthogonal - Y_i
- C. Use Modified Gram-Schmidt, Twice, to Make Block of Vectors, Y_i , Orthogonal to all Previously Calculated Vectors - V_i

STEP 3. MAKE VECTORS STIFFNESS ORTHOGONAL

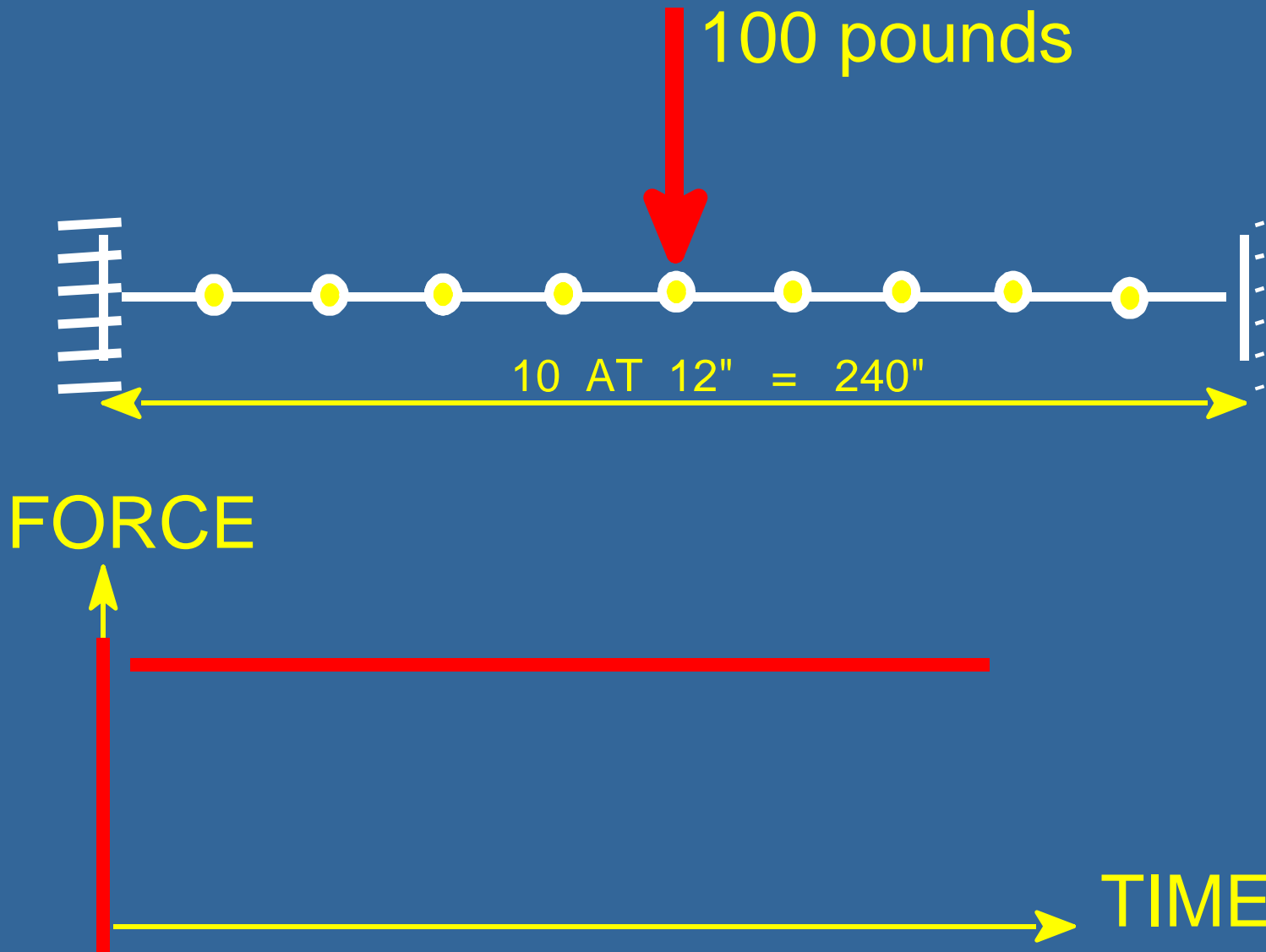
A. SOLVE Nb x Nb Eigenvalue Problem

$$[V^T K V] Z = [\omega^2] Z$$

B. CALCULATE MASS AND STIFFNESS
ORTHOGONAL **LDR** VECTORS

$$V_R = V Z = \Phi$$

DYNAMIC RESPONSE OF BEAM



MAXIMUM DISPLACEMENT

Number of Vectors Vectors		Eigen Vectors		Load Dependent	
1	0.004572	(-2.41)	0.004726	(+0.88)	
2	0.004572	(-2.41)	0.004591	(-2.00)	
3	0.004664	(-0.46)	0.004689	(+0.08)	
4	0.004664	(-0.46)	0.004685	(+0.06)	
5	0.004681	(-0.08)	0.004685	(0.00)	
7	0.004683	(-0.04)			
9	0.004685	(0.00)			

(Error in Percent)

MAXIMUM MOMENT

Number of Vectors Vectors		Eigen Vectors		Load Dependent	
1	4178	(- 22.8 %)	5907	(+ 9.2)	
2	4178	(- 22.8)	5563	(+ 2.8)	
3	4946	(- 8.5)	5603	(+ 3.5)	
4	4946	(- 8.5)	5507	(+ 1.8)	
5	5188	(- 4.1)	5411	(0.0)	
7	5304	(- .0)			
9	5411	(0.0)			

(Error in Percent)

Push Over Analysis

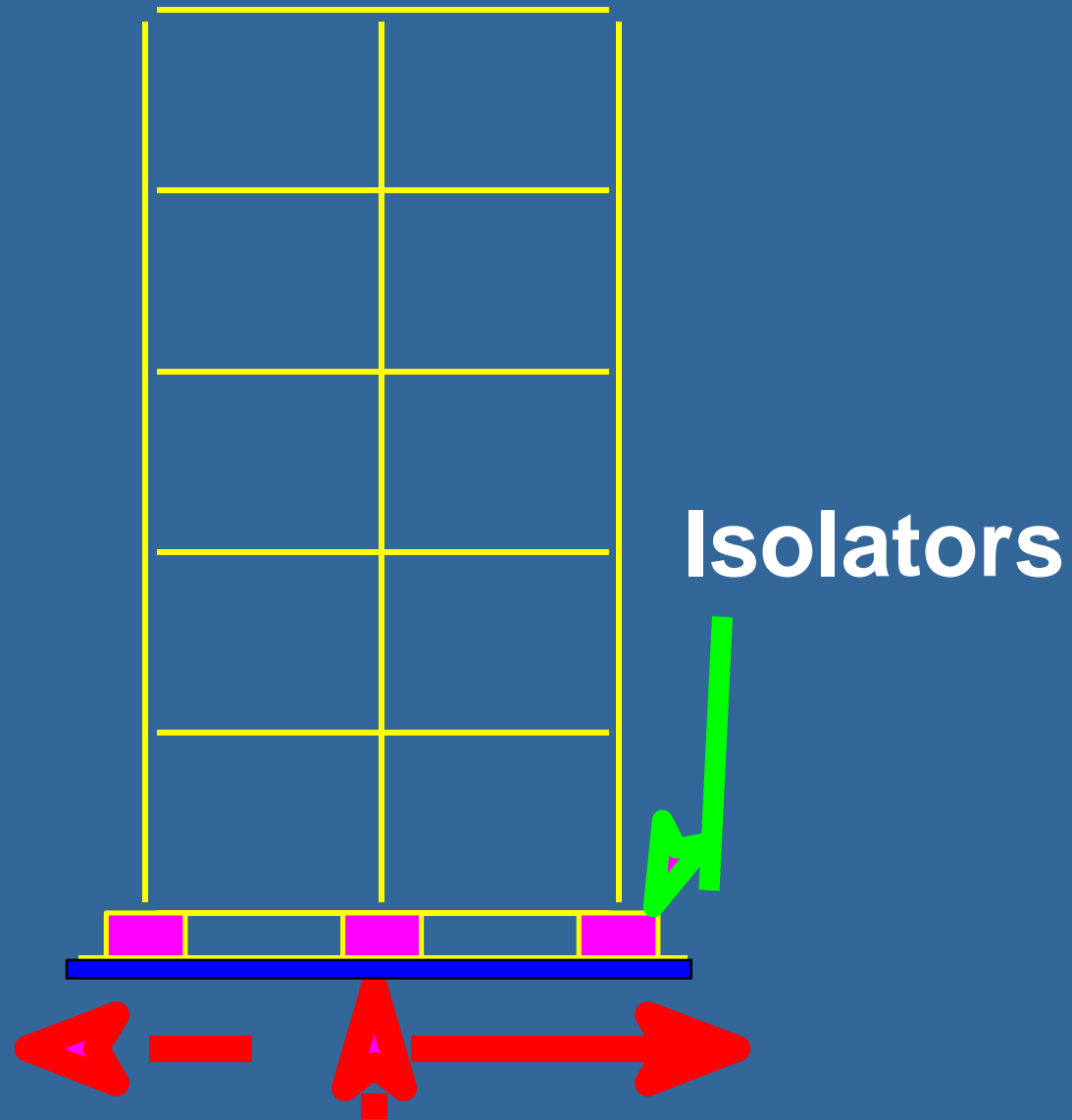
1. One-dimensional Static Loads
2. No Energy Dissipation
3. Inertia Forces Not Considered
4. Defines One Failure Mode
5. Higher Mode Effects Neglected

FAST NONLINEAR ANALYSIS

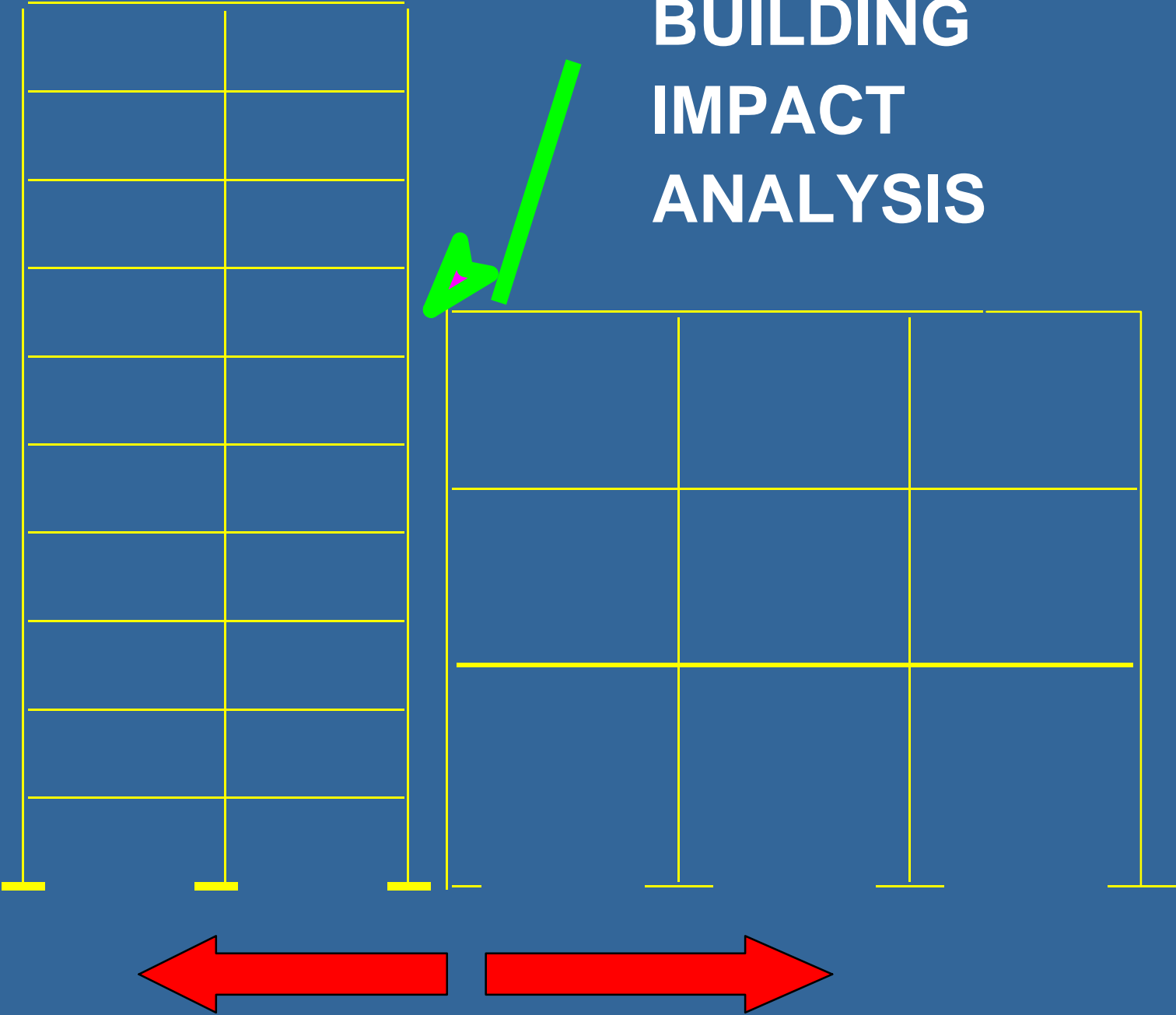
1. EVALUATE **LDR** VECTORS WITH NONLINEAR ELEMENTS REMOVED AND DUMMY ELEMENTS ADDED FOR STABILITY
2. SOLVE ALL MODAL EQUATIONS WITH NONLINEAR FORCES ON THE RIGHT HAND SIDE
3. USE EXACT INTEGRATION WITHIN EACH TIME STEP
4. FORCE AND ENERGY EQUILIBRIUM ARE SATISFIED AT EACH TIME STEP BY ITERATION

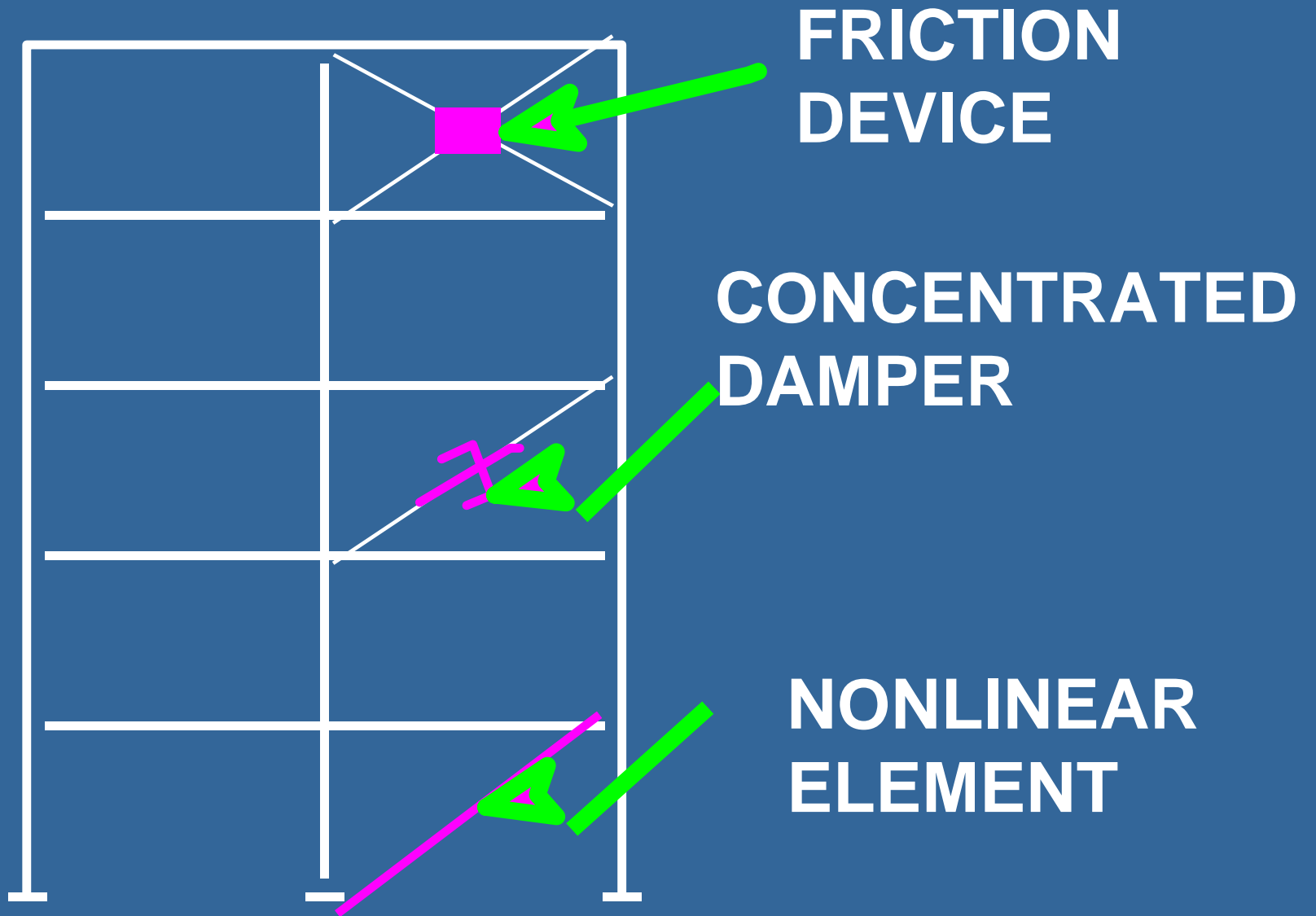
*The FNA Method Is Designed For
The Static And Dynamic Analysis
Of Nonlinear Structures
With A Limited Number Of
Predefined Nonlinear Elements*

BASE ISOLATION

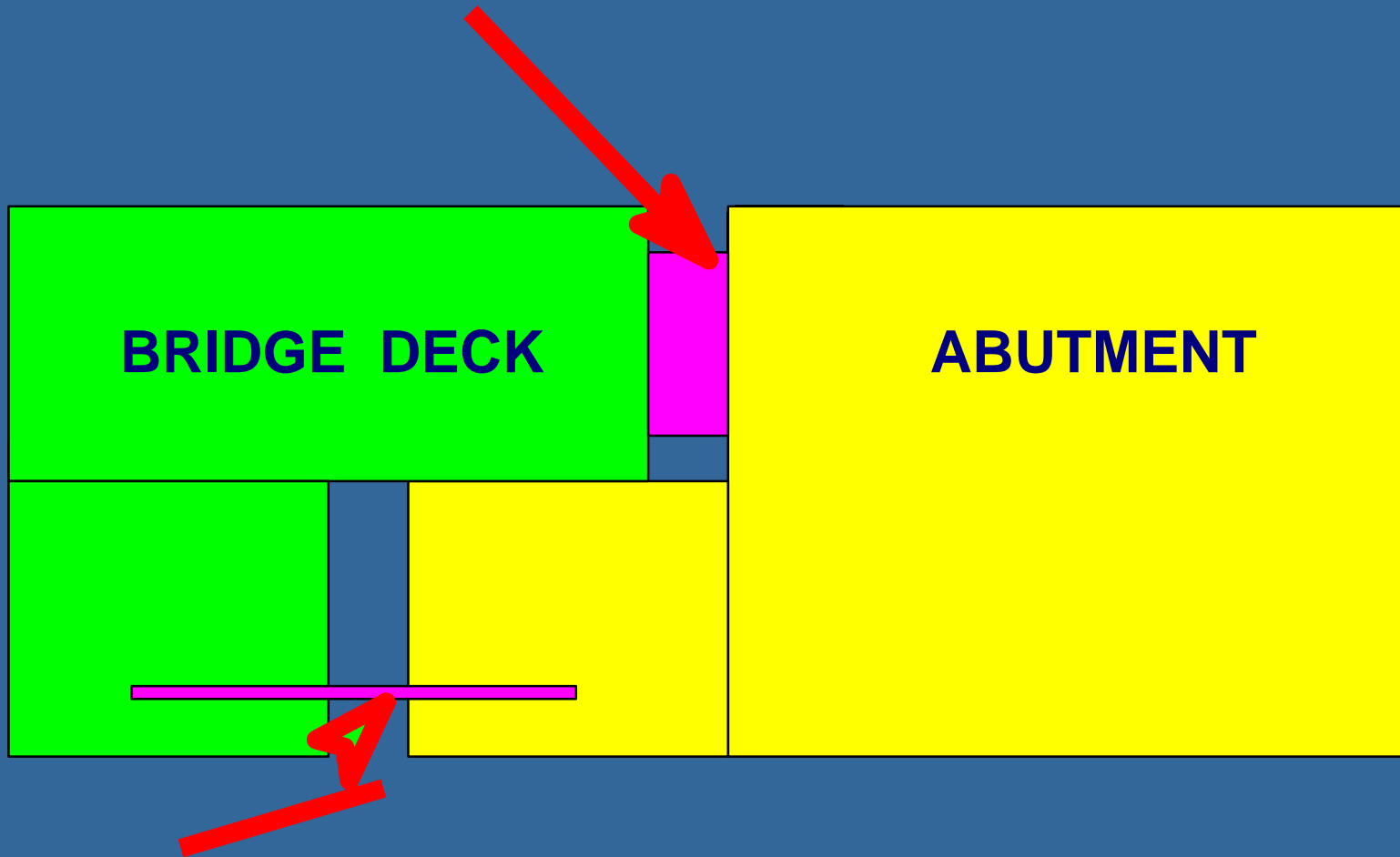


BUILDING IMPACT ANALYSIS





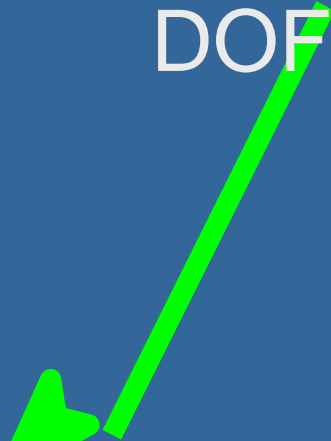
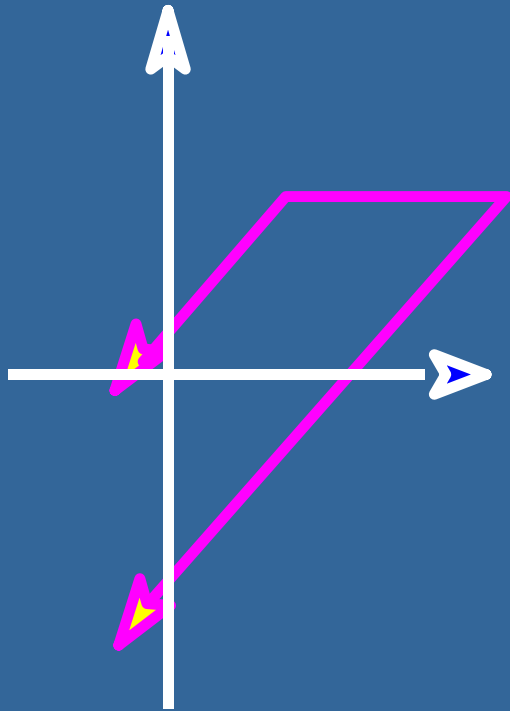
GAP ELEMENT



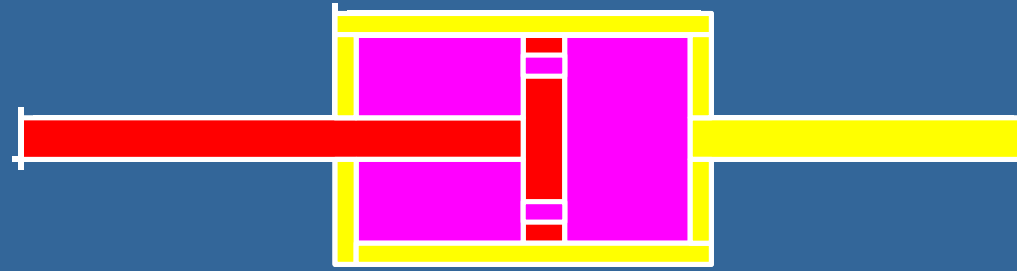
TENSION ONLY ELEMENT

PLASTIC HINGES

2 ROTATIONAL
DOF

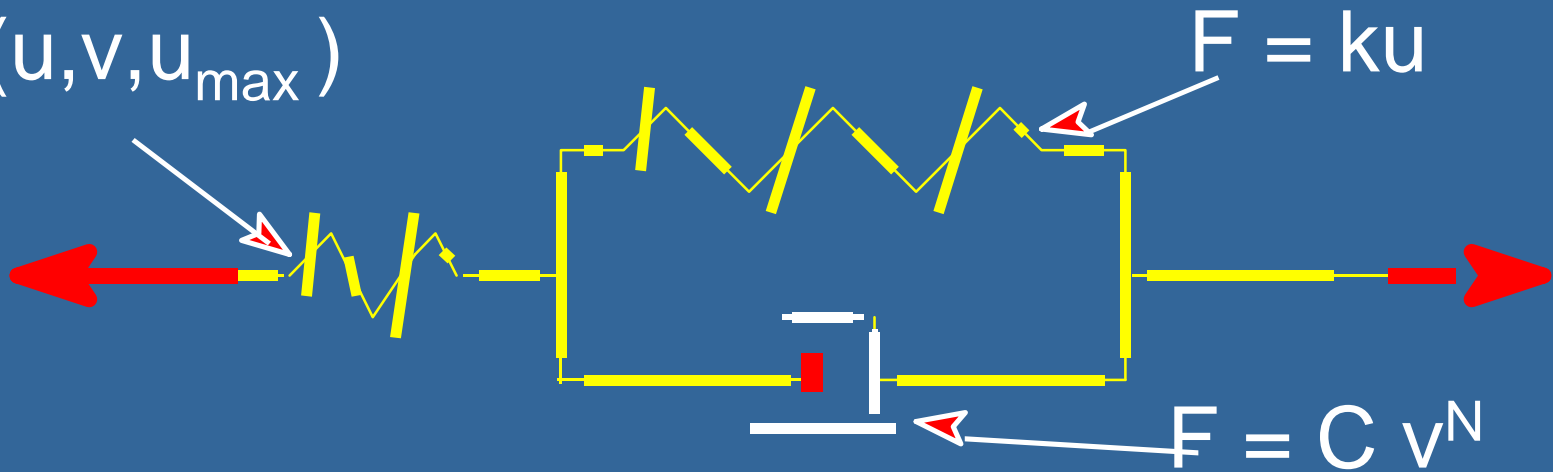


DEGRADING STIFFNESS ?



Mechanical Damper

$$F = f(u, v, u_{\max})$$



Mathematical Model

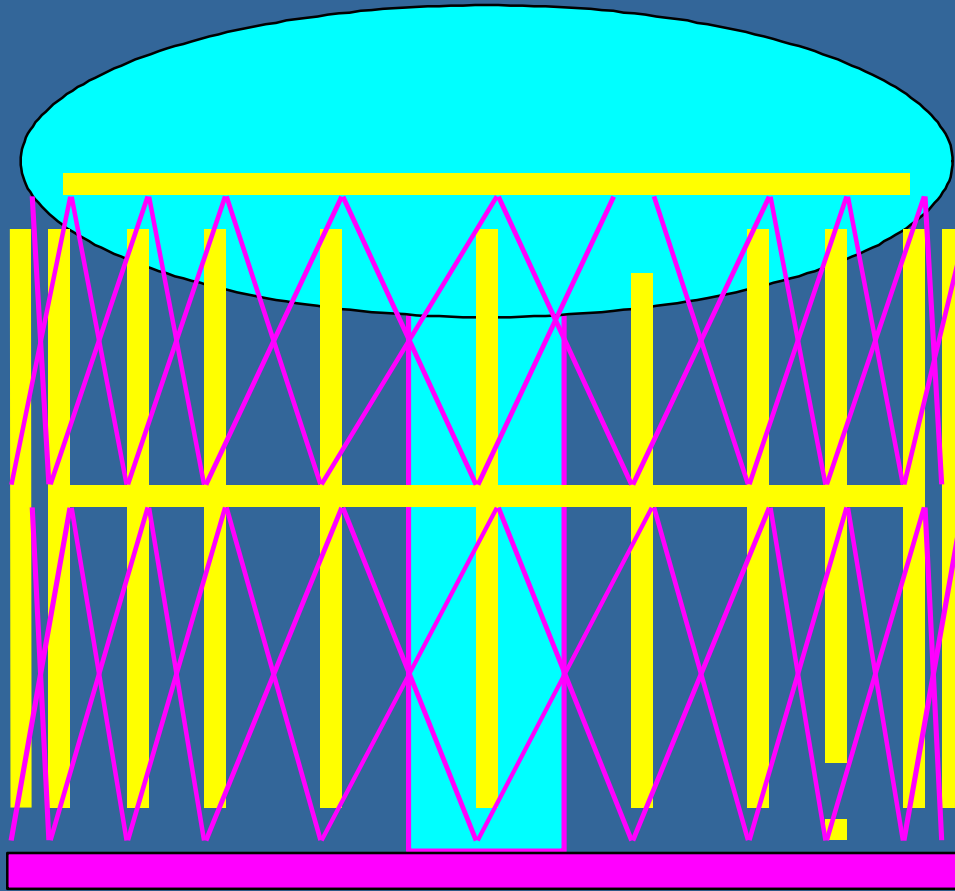
LINEAR VISCOUS DAMPING

DOES NOT EXIST IN NORMAL STRUCTURES AND FOUNDATIONS

5 OR **10** PERCENT MODAL DAMPING VALUES ARE OFTEN USED TO JUSTIFY ENERGY DISSIPATION DUE TO NONLINEAR EFFECTS

IF ENERGY DISSIPATION DEVICES ARE USED THEN **1** PERCENT MODAL DAMPING SHOULD BE USED FOR THE ELASTIC PART OF THE STRUCTURE - **CHECK ENERGY PLOTS**

103 FEET DIAMETER - 100 FEET HEIGHT



NONLINEAR
DIAGONALS

BASE
ISOLATION

ELEVATED WATERSTORAGE TANK

COMPUTER MODEL

92 NODES

103 ELASTIC FRAME ELEMENTS

56 NONLINEAR DIAGONAL ELEMENTS

600 TIME STEPS @ 0.02 Seconds

COMPUTER TIME REQUIREMENTS

PROGRAM

ANSYS INTEL 3 Days (4300 Minutes)
486

ANSYS CRAY 3 Hours (180 Minutes)

SADSAP INTEL 486 2 Minutes

(B Array was 56 x 20)

Nonlinear Equilibrium Equations

$$M a + C v + K u + F_N = F$$

Or

$$M a + C v + K u = F - F_N$$

Where

$F_N =$ The Global Node Loads due to the Forces in the Nonlinear Elements

Nonlinear Equilibrium Equations

$$M a + C v + [K + k_E] u = F - F_N + k_E u$$

Where

$k_E =$ The Effective Linear Stiffness
of the Nonlinear Elements are of
arbitrary values for zero damping

Summary Of FNA Method

1. Calculate Ritz Vectors for Structure With the Nonlinear Elements Removed.
2. These Vectors Satisfy the Following Orthogonality Properties

$$f^T K f = \Omega^2 \quad f^T M f = I$$

3. The Solution Is Assumed to Be a Linear Combination of the LDR Vectors. Or,

$$u(t) = f Y(t) = \sum_n f_n y(t)_n$$

Which Is the Standard
Mode Superposition Equation

Remember the LDR Vectors Are a Linear Combination of the Exact Eigenvectors; Plus, the Static Displacement Vectors.

No Additional Approximations Are Made.

4. A typical modal equation is uncoupled. However, the modes are coupled by the unknown nonlinear modal forces which are of the following form:

$$f_n = \mathbf{f}_n F_n$$

5. The deformations in the nonlinear elements can be calculated from the following displacement transformation equation:

$$d = A u$$

6. Since $u(t) = f Y(t)$ the deformations in the nonlinear elements can be expressed in terms of the modal response by

$$d(t) = A f Y(t) = B Y(t)$$

Where the size of the B array is equal to the number of deformations times the number of LDR vectors.

The B array is calculated only once prior to the start of mode integration.

THE B ARRAY CAN BE STORED IN RAM

7. The nonlinear element forces are calculated, for iteration i , at the end of each time step t

$\mathbf{d}_t^{(i)} = \mathbf{B}\mathbf{Y}_t^{(i)}$ = Deformations in

Nonlinear Elements

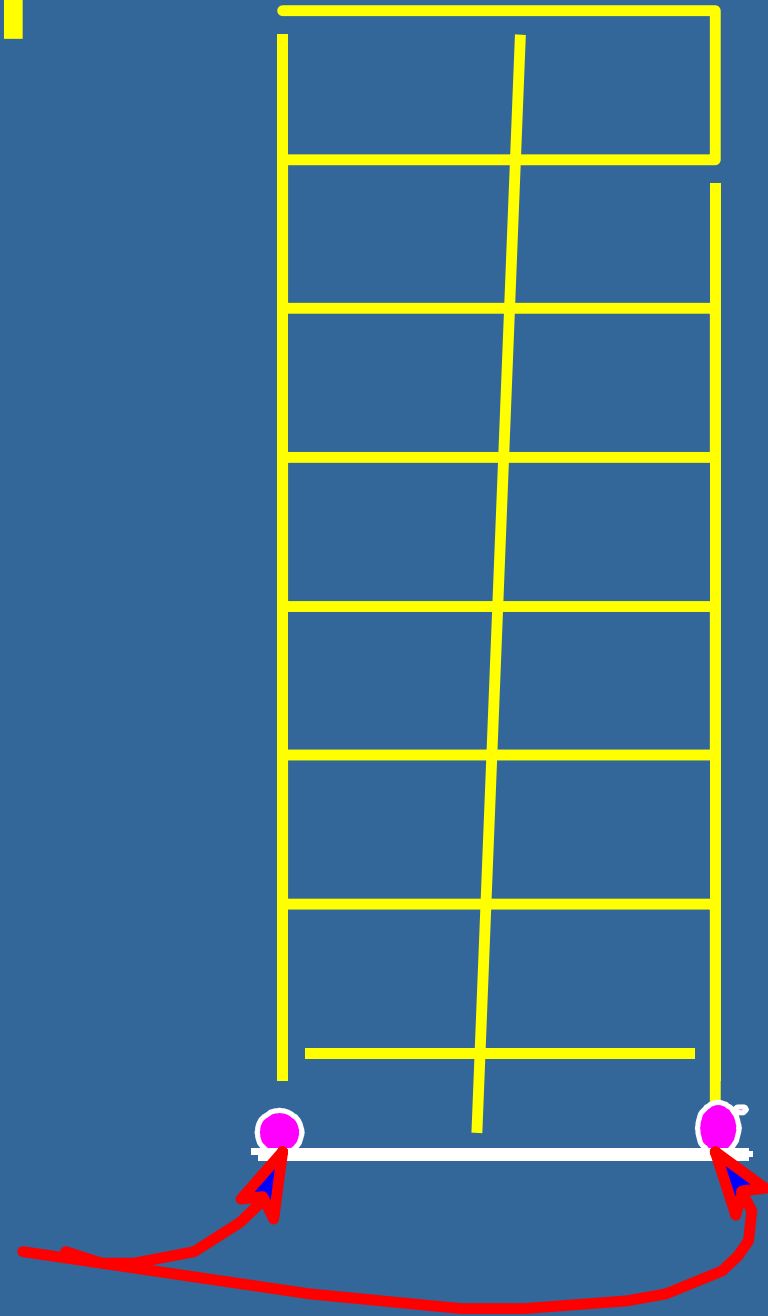
$\mathbf{P}_t^{(i)}$ = Function of Element History

$\mathbf{f}_{N_t}^{(i)} = \mathbf{B}^T \mathbf{Y}_t^{(i)}$ = Nonlinear Modal Loads

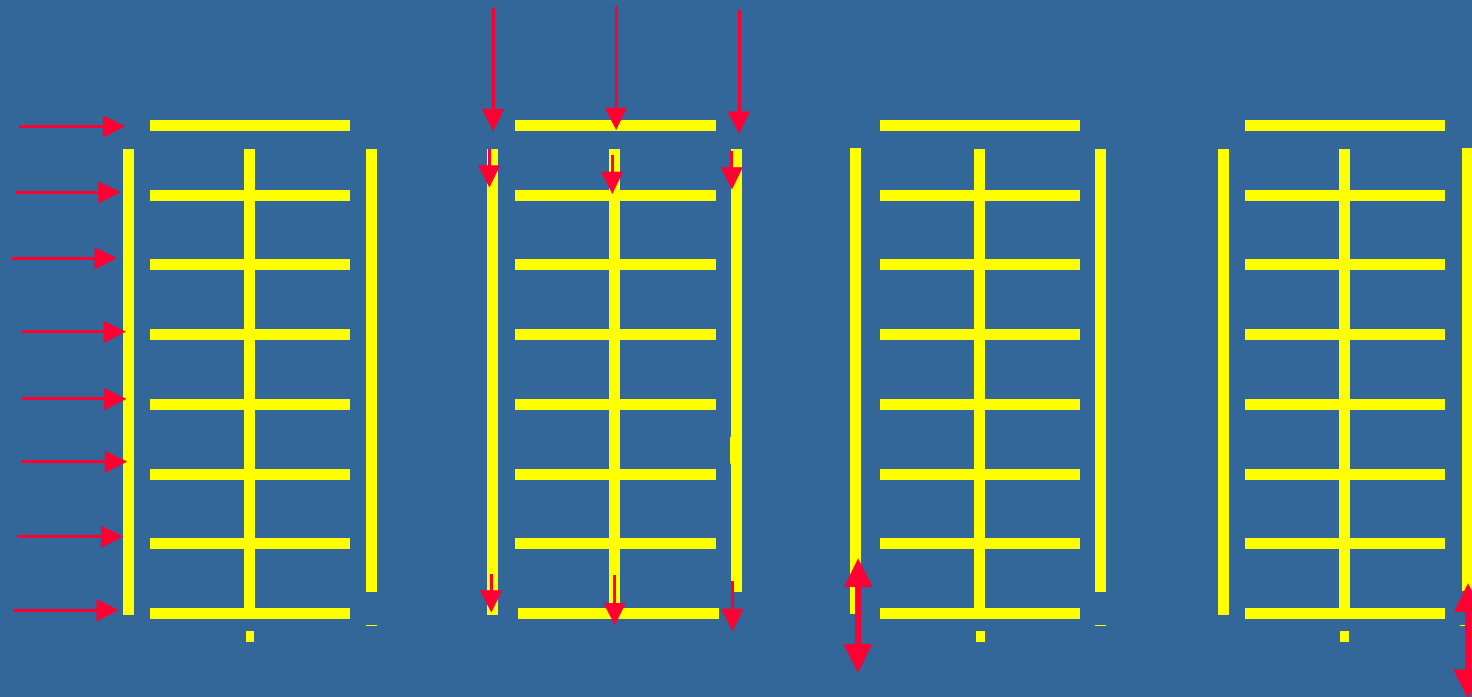
$\mathbf{Y}_t^{(i+1)}$ = New Solution of Modal Equation

FRAME WITH UPLIFTING ALLOWED

UPLIFTING
ALLOWED



Four Static Load Conditions Are Used To Start The Generate of LDR Vectors



EQ

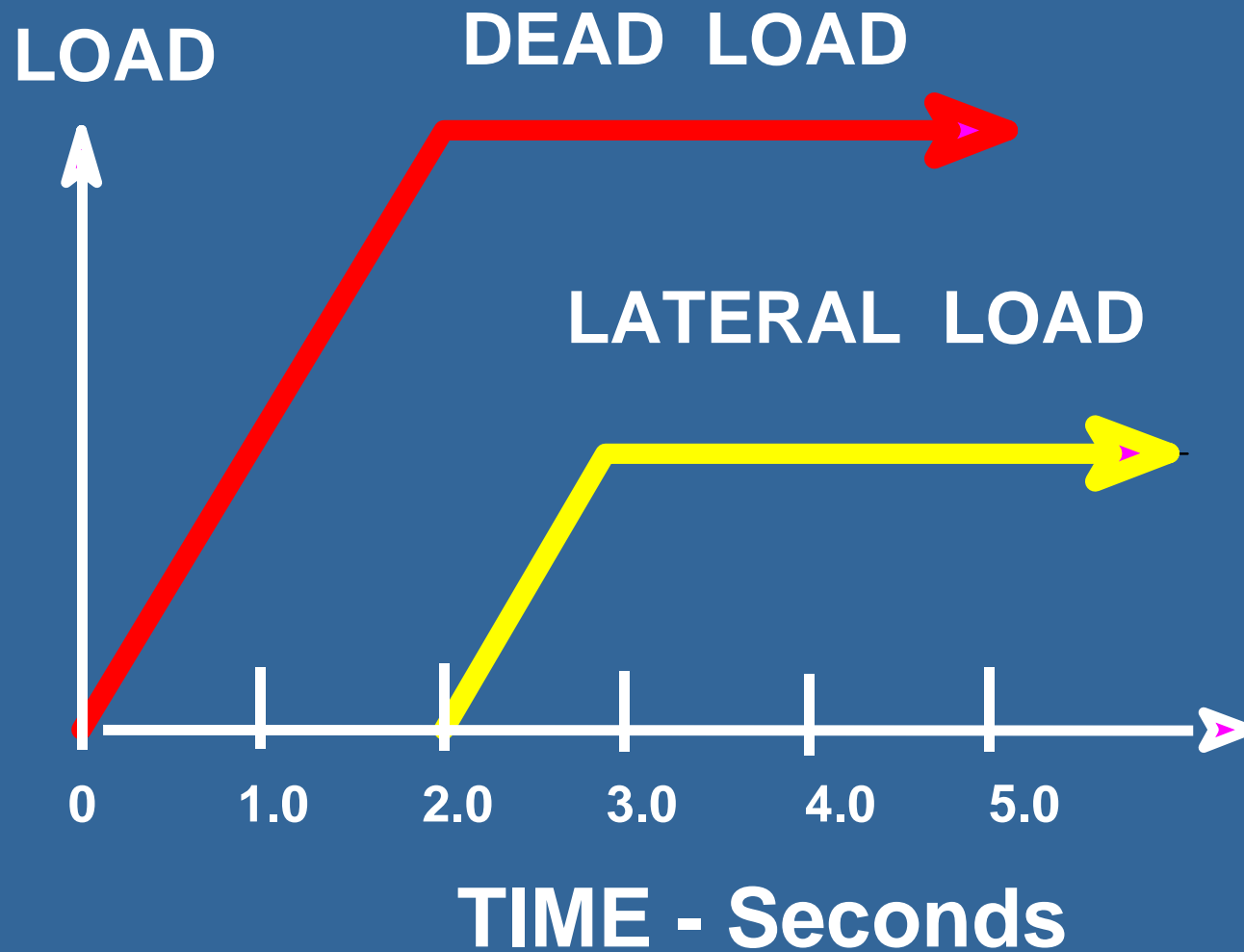
DL

Left

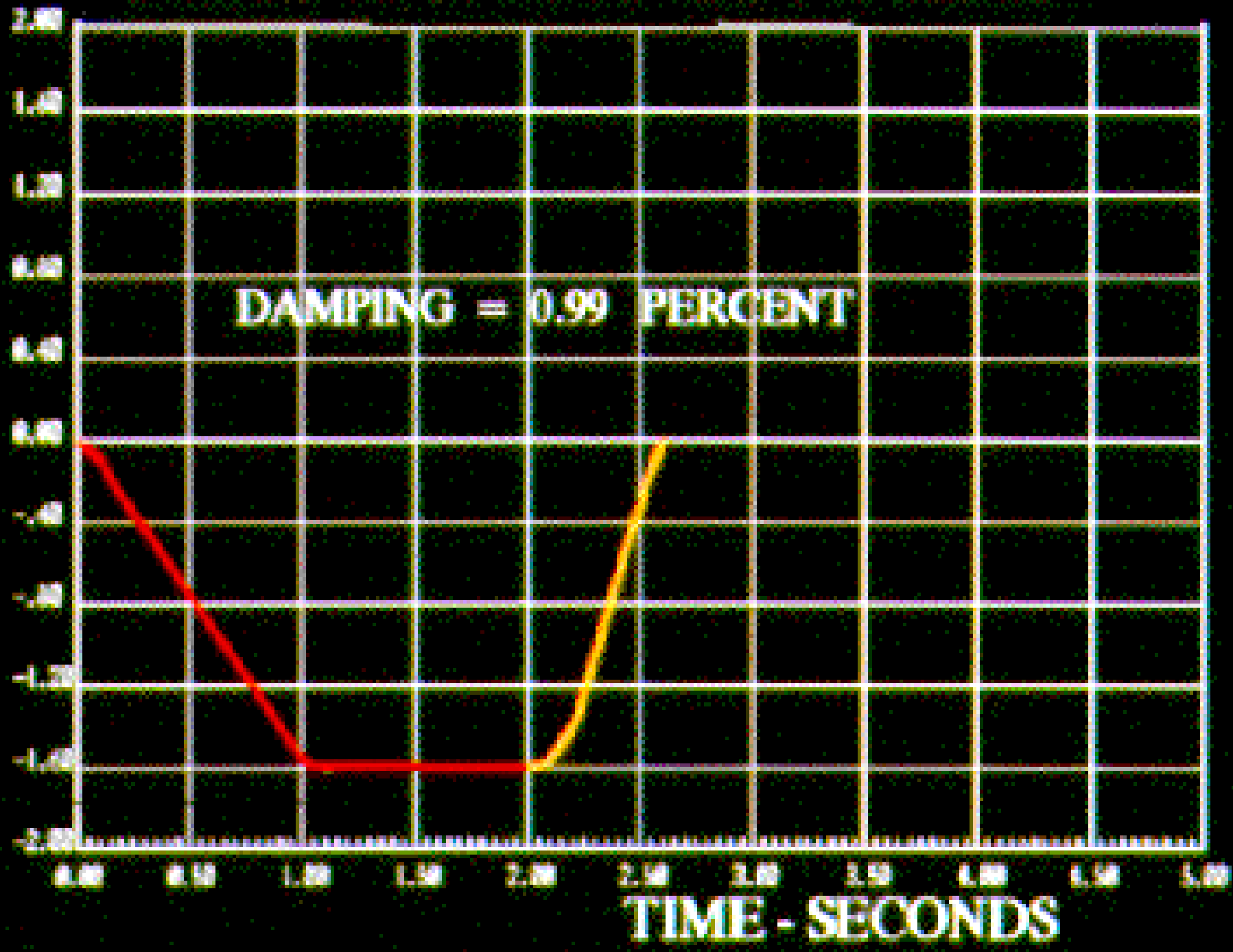
Right

NONLINEAR STATIC ANALYSIS

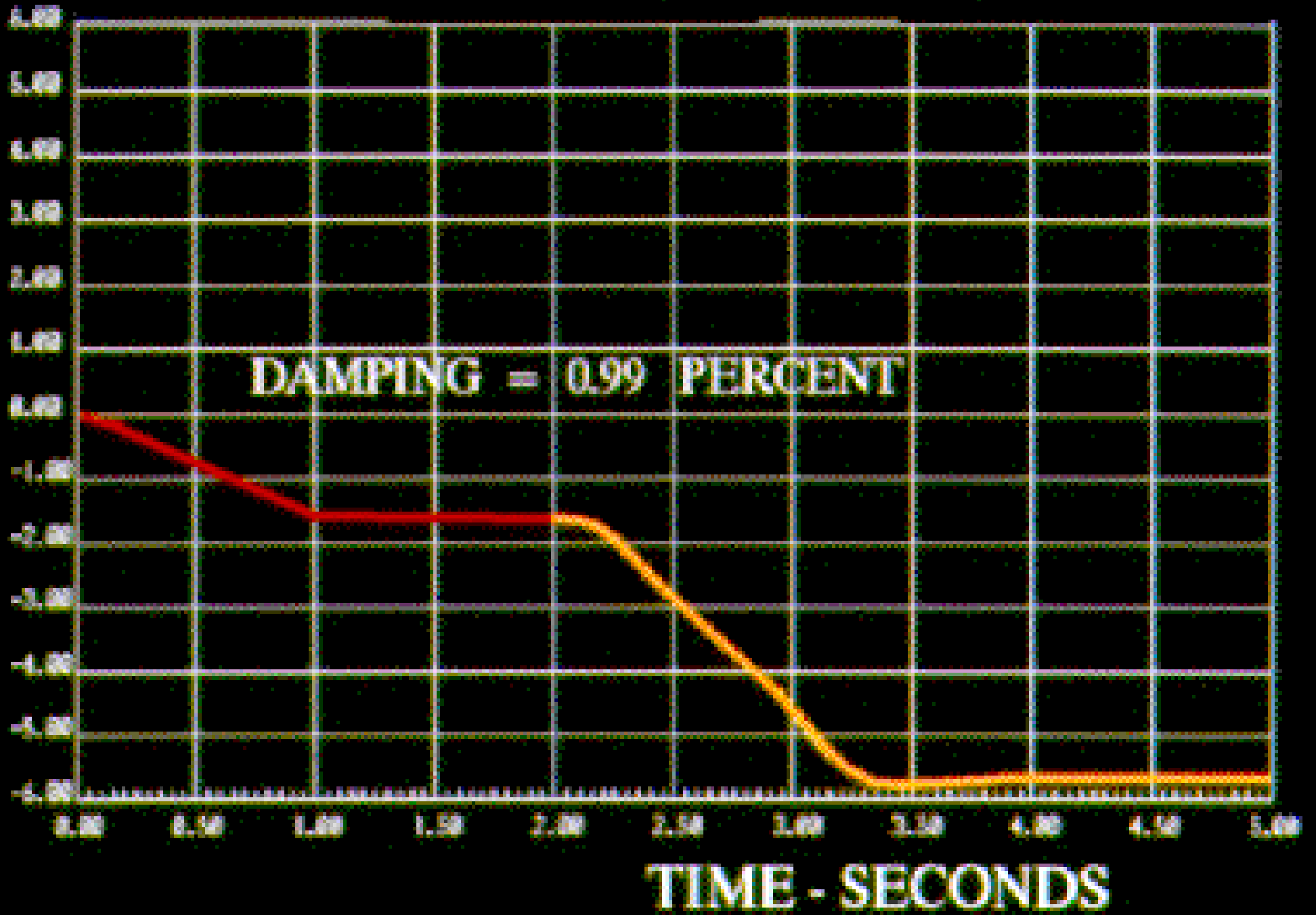
50 STEPS AT $\Delta T = 0.10$ SECONDS



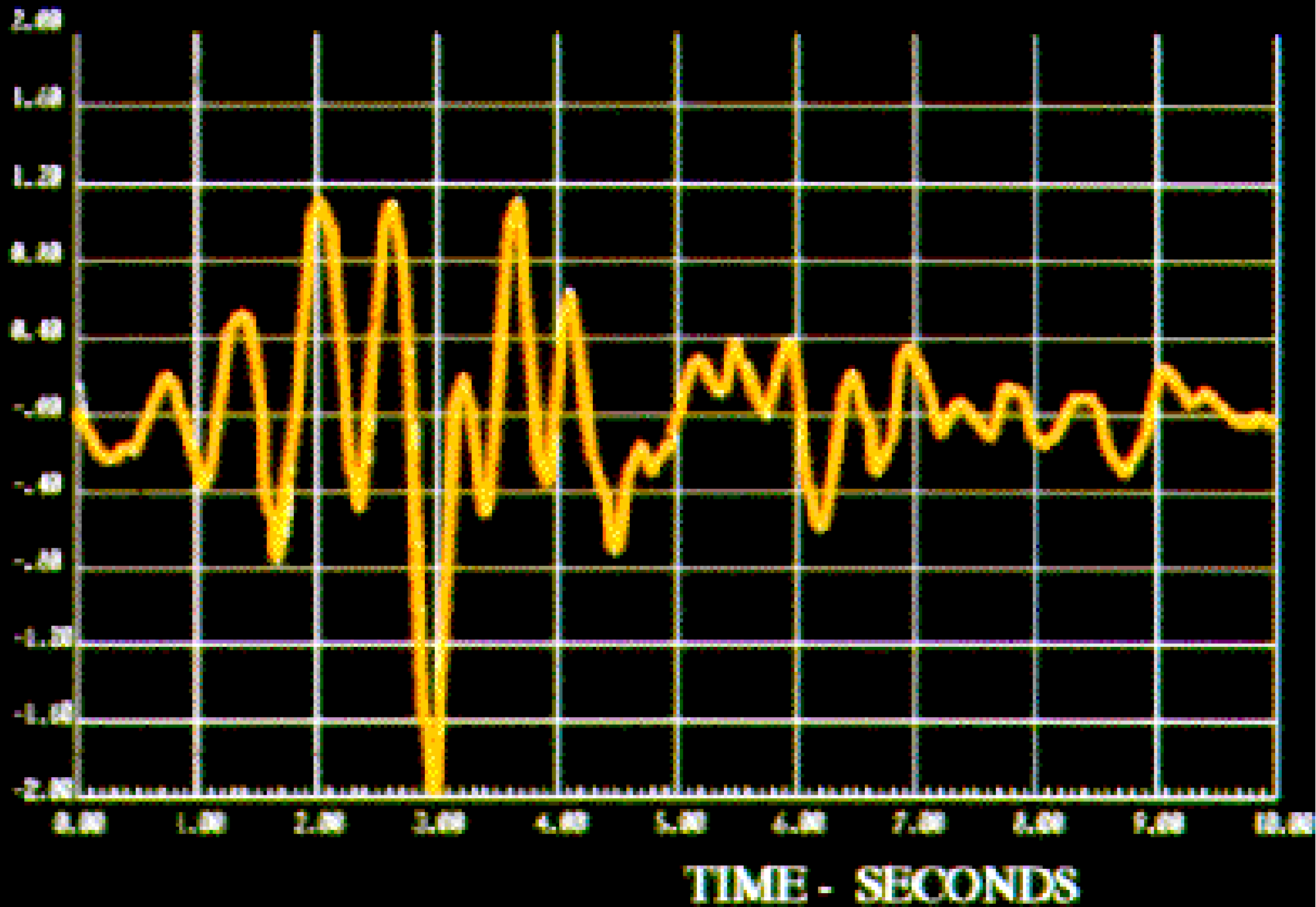
FORCE AT BASE OF LEFT COLUMN



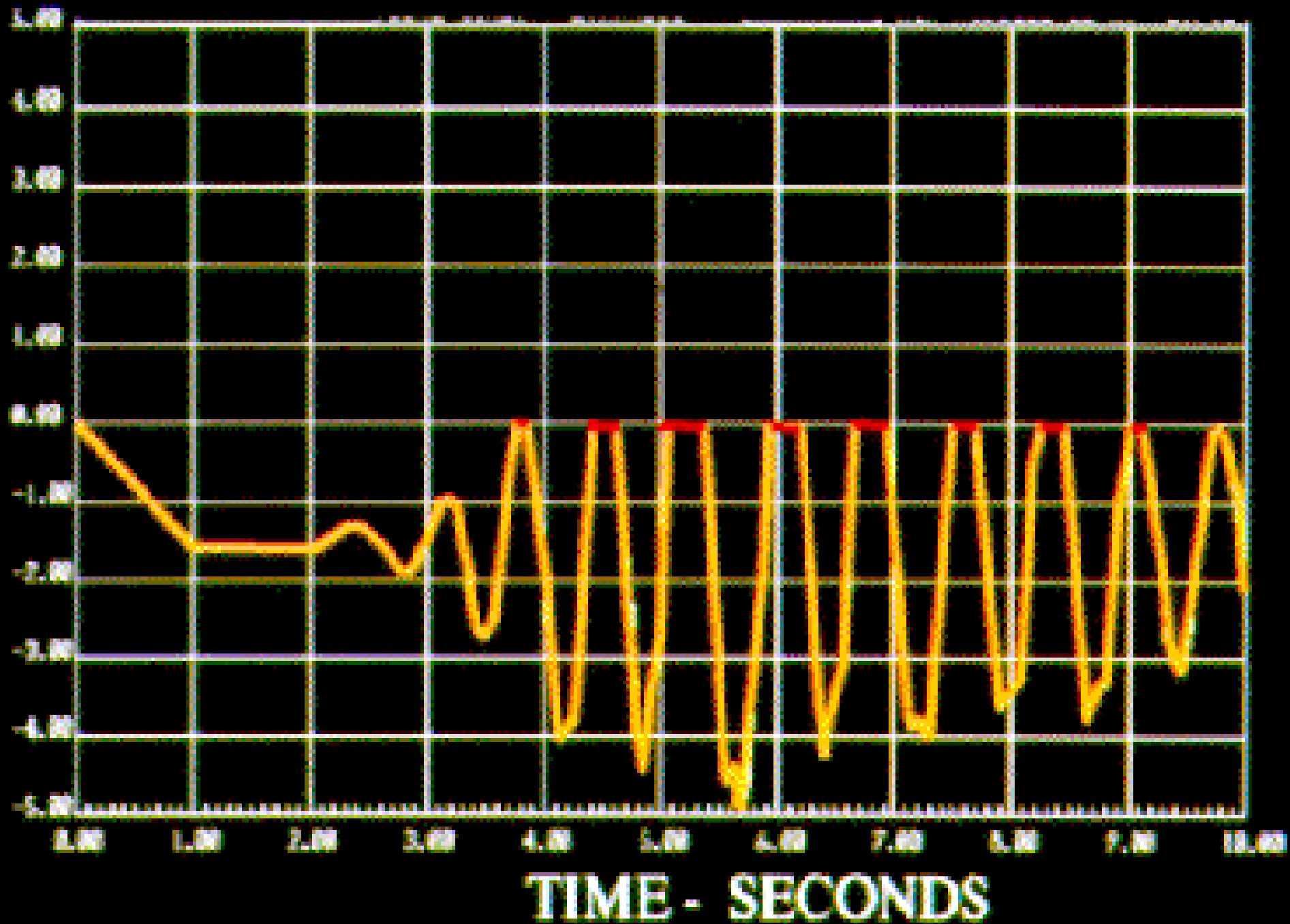
FORCE AT BASE OF RIGHT COLUMN



COMPONENT OF LOMA PRIETA EARTHQUAKE



AXIAL FORCE AT BASE OF LEFT COLUMN



Advantages Of The FNA Method

1. The Method Can Be Used For Both Static And Dynamic Nonlinear Analyses
2. The Method Is Very Efficient And Requires A Small Amount Of Additional Computer Time As Compared To Linear Analysis
2. The Method Can Easily Be Incorporated Into Existing Computer Programs For LINEAR DYNAMIC ANALYSIS.

FUTURE DEVELOPMENTS FOR SAP2000

1. ADDITIONAL NONLINEAR ELEMENTS
crush and yield elements
degrading stiffness elements
general CABLE element
2. SUBSTRUCTURE OPTION
3. SOIL STRUCTURE INTERACTION
4. FLUID - STRUCTURE INTERACTION
5. ADDITIONAL DOCUMENTATION
AND EXAMPLES

EXAMPLE ON THE USE OF SUBSTRUCTURE ANALYSIS

LINEAR AND NONLINEAR ANALYSIS

OF THE

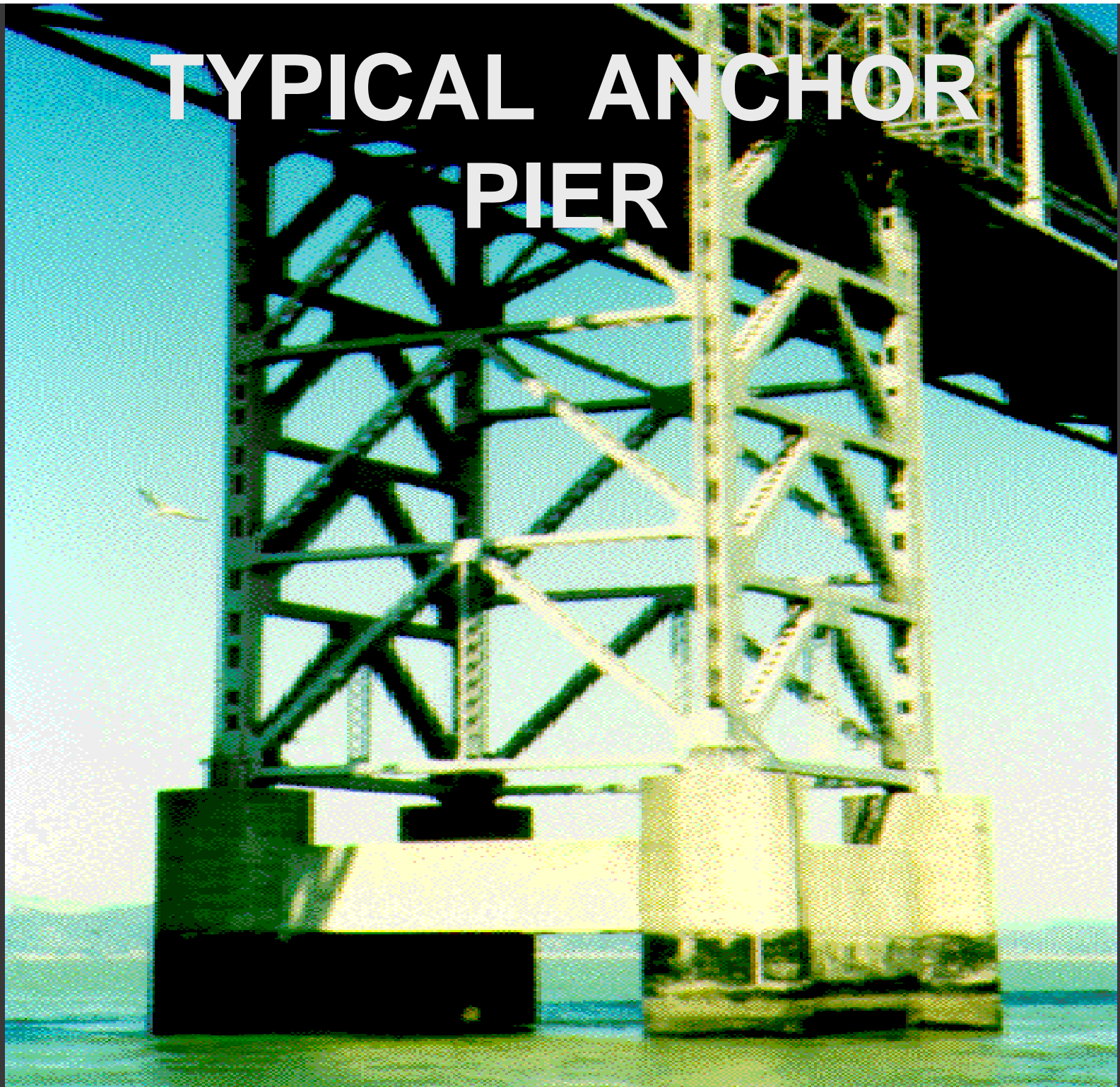
RICHMOND-SAN RAFAEL BRIDGE



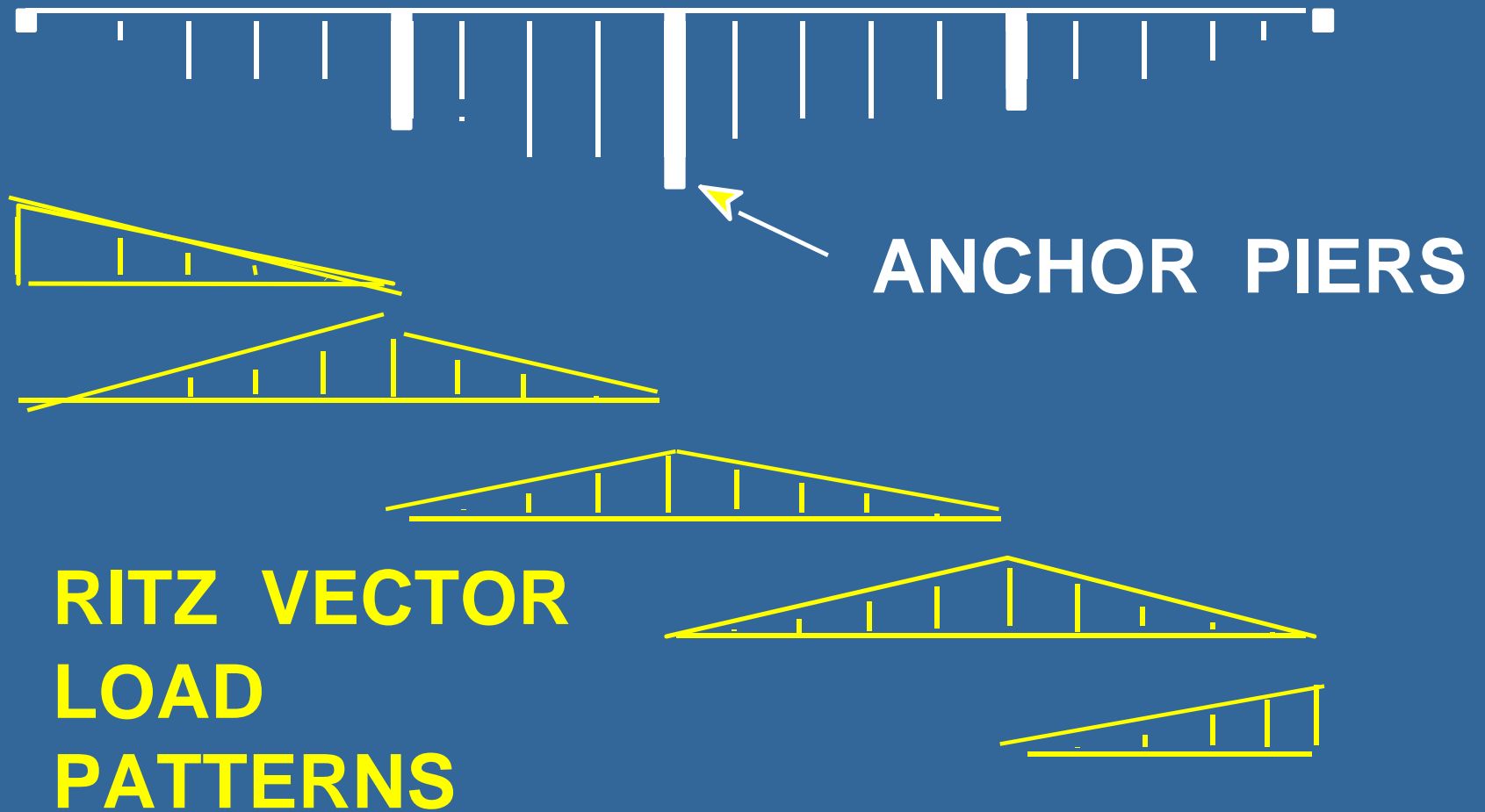




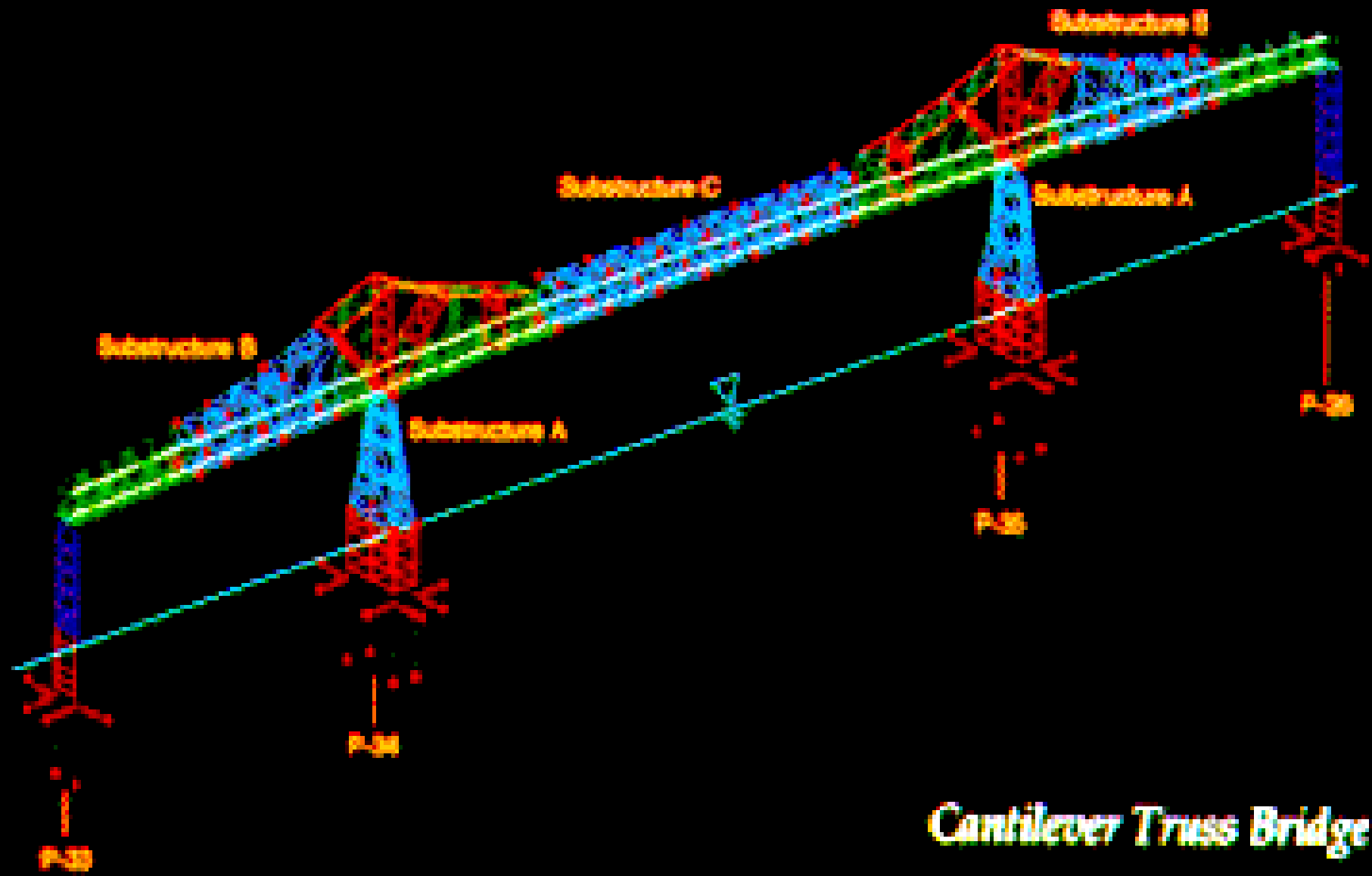
TYPICAL ANCHOR PIER



MULTISUPPORT ANALYSIS (Displacements)

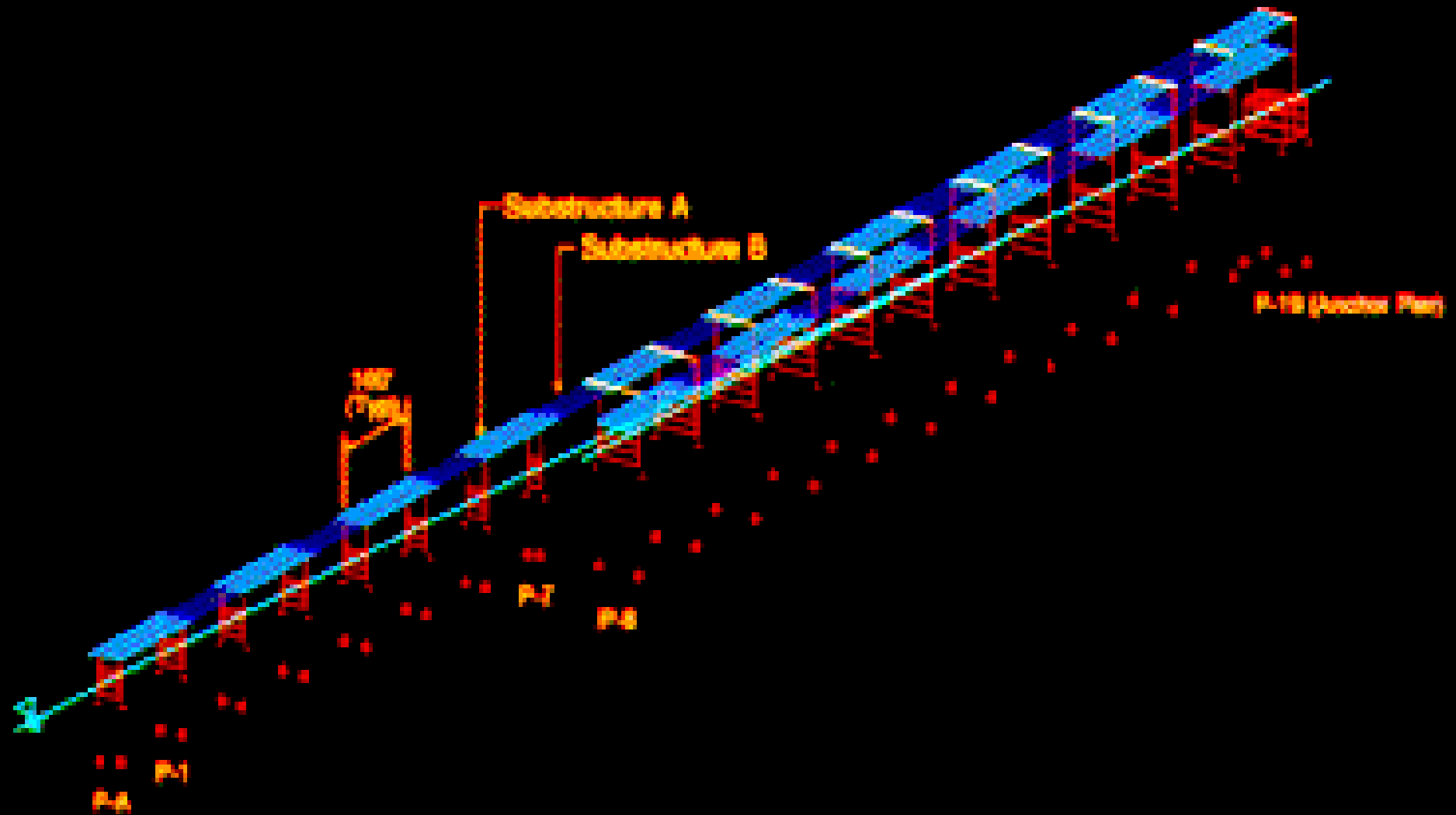


Sub-Structured Model



Cantilever Truss Bridge

Richmond-San Rafael Bridge



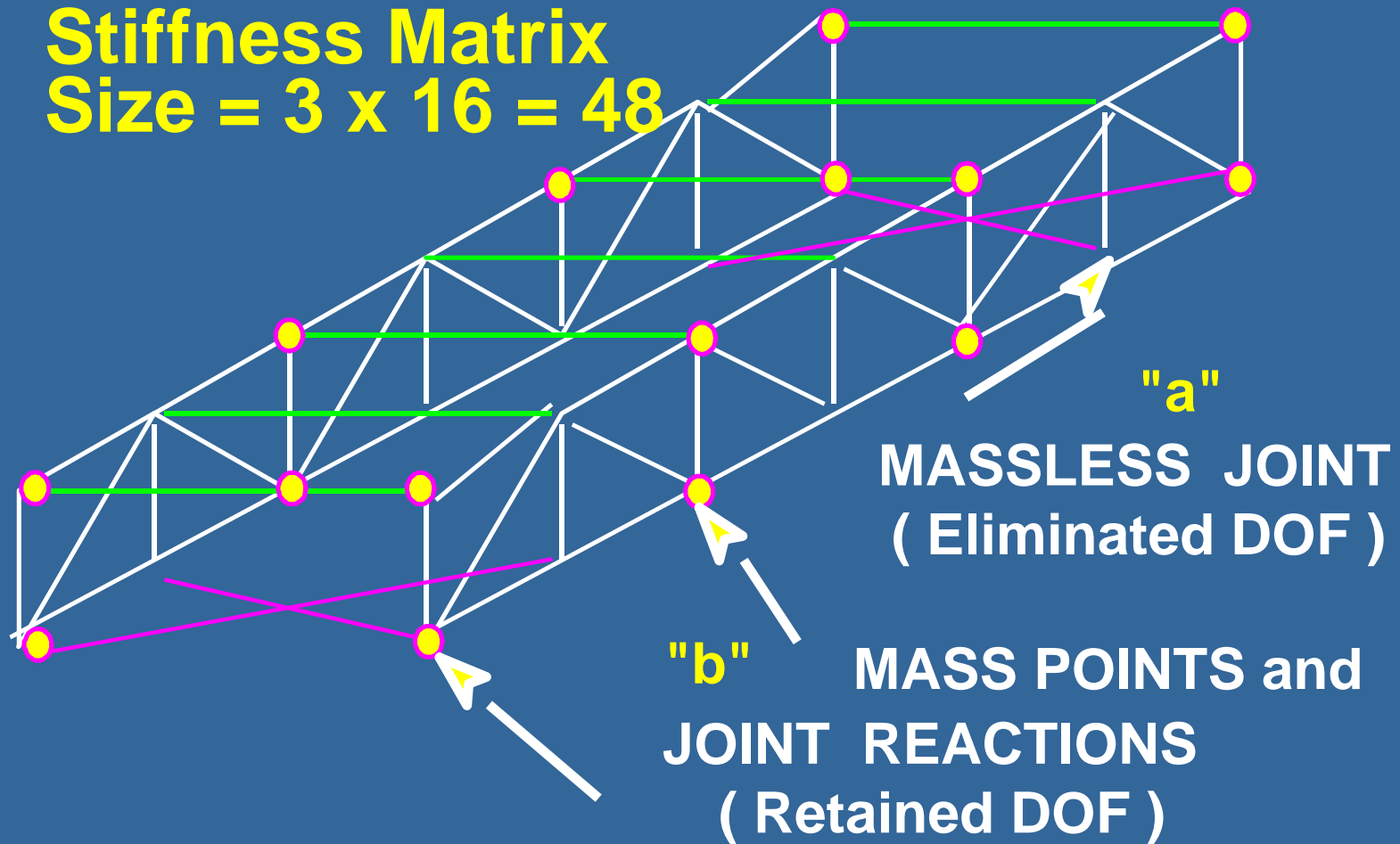
Seg-A/Plate Girder Spans

◆ HCF KAMMER



SUBSTRUCTURE PHYSICS

Stiffness Matrix
Size = $3 \times 16 = 48$



SUBSTRUCTURE SUBROUTINE

$$\left[\begin{array}{c|c} k_{aa} & k_{ab} \\ \hline k_{ba} & k_{bb} \end{array} \right]$$

SEE FORTRAN LISTING

ADVANTAGES IN THE USE OF SUBSTRUCTURES

- 1. FORM OF MESH GENERATION**
- 2. LOGICAL SUBDIVISION OF WORK**
- 3. MANY SHORT COMPUTER RUNS**
- 4. RERUN ONLY SUBSTRUCTURES
WHICH WERE REDESIGNED**
- 5. PARALLEL POST PROCESSING
USING NETWORKING**

RICHMOND - SAN RAFAEL BRIDGE

RETROFIT

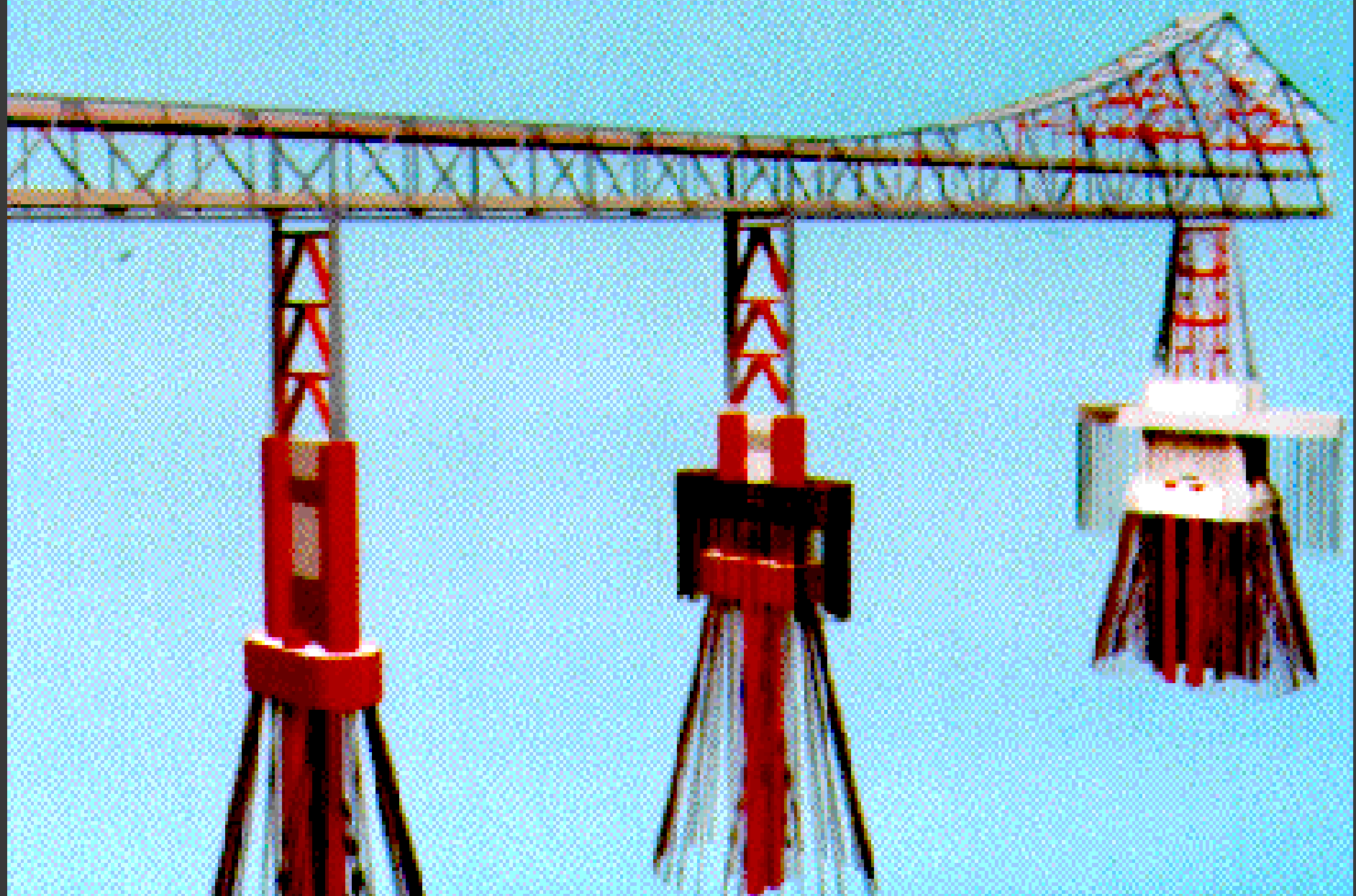


RICHMOND - SAN RAFAEL BRIDGE AS BUILT

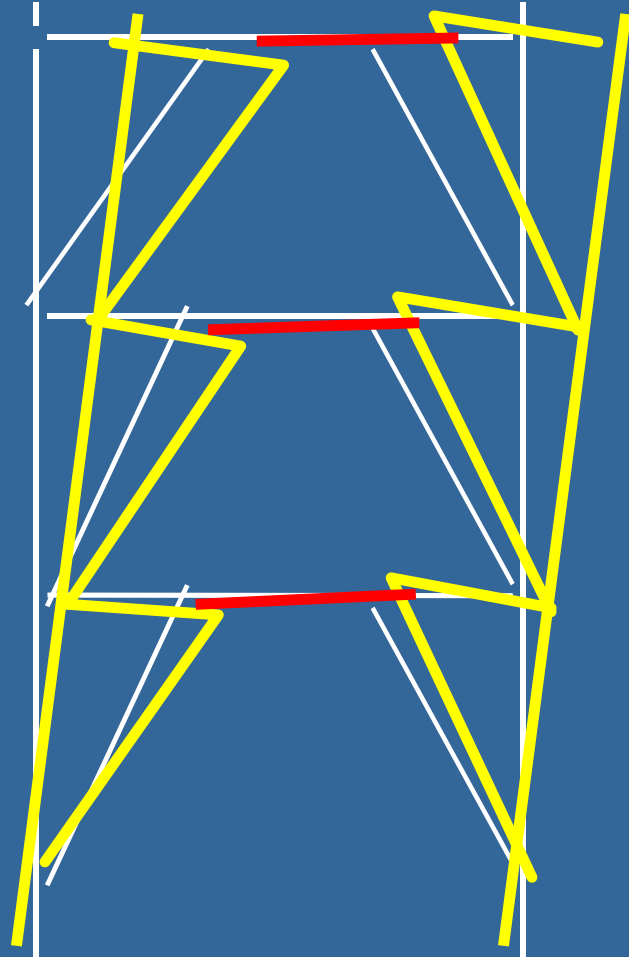
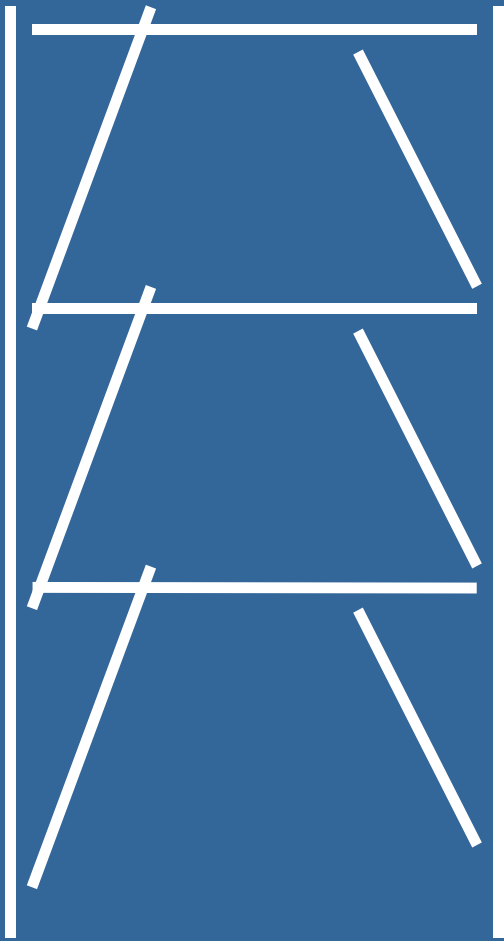


RICHMOND - SAN RAFAEL BRIDGE

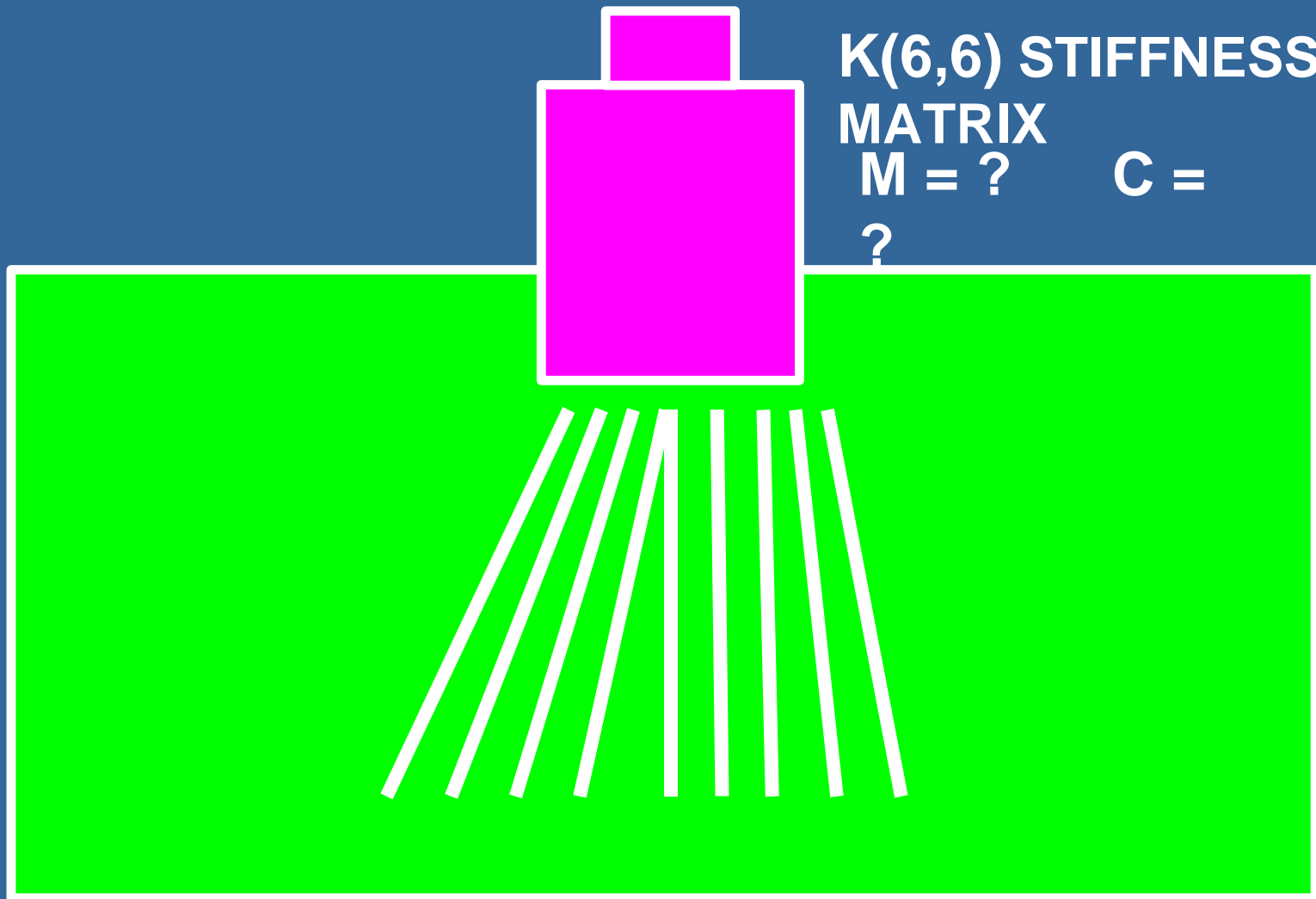
RETROFIT ELEMENTS



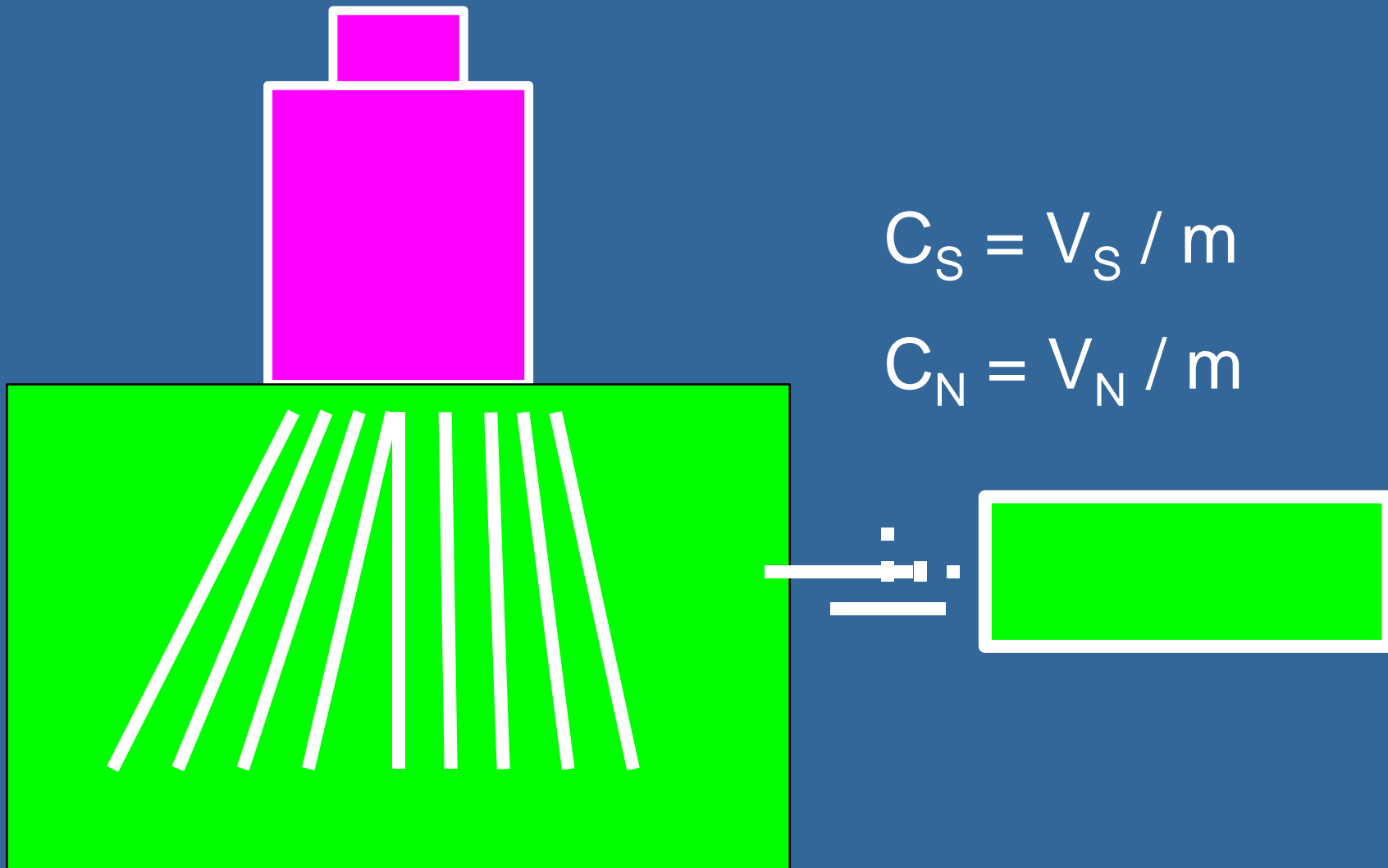
ECCENTRICALLY BRACED FRAME



EFFECTIVE LINEAR MODEL OF FOUNDATION PILE GROUP



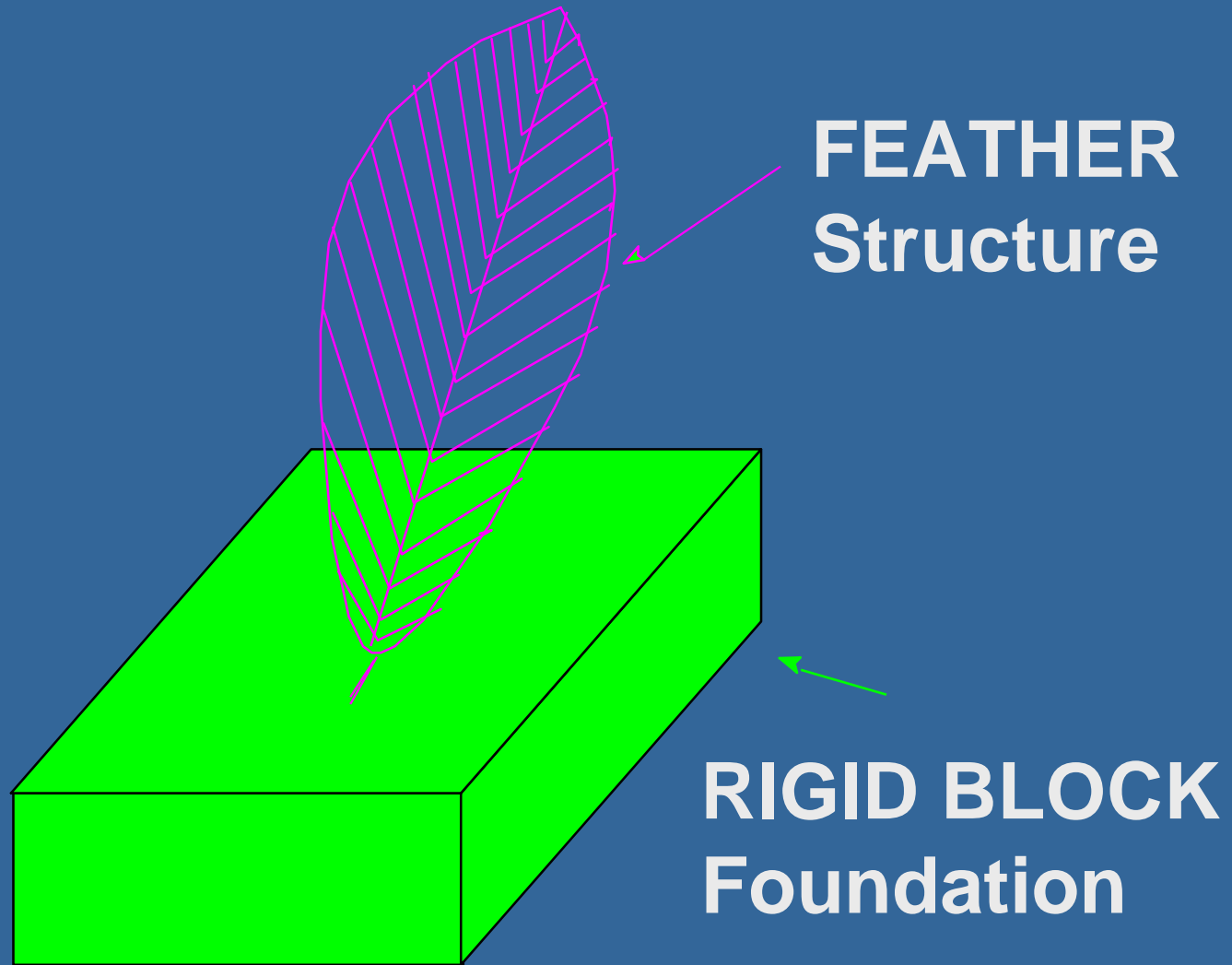
NONLINEAR MODEL OF FOUNDATION PILE GROUP ??



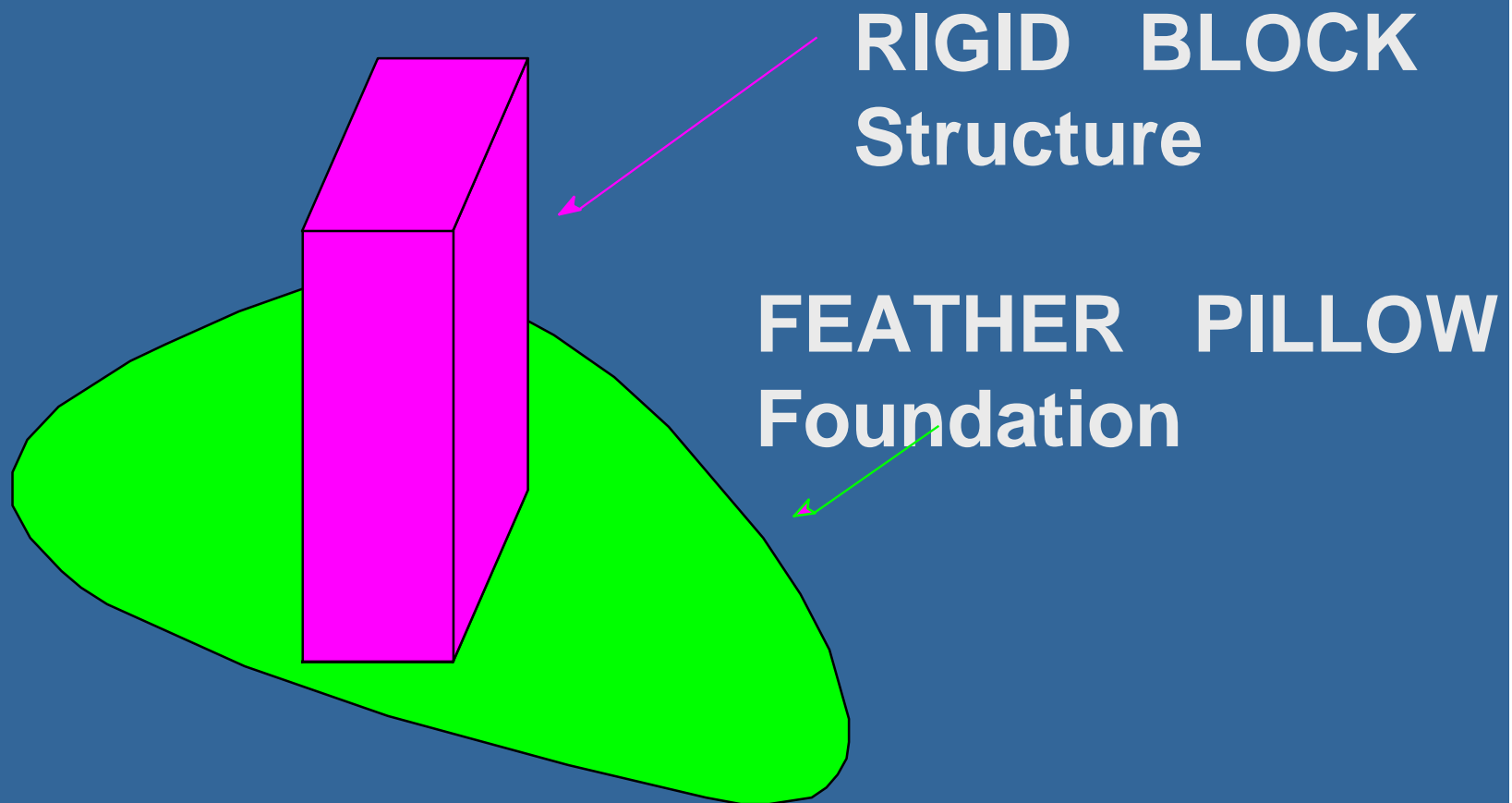
SITE ANALYSIS - SHAKE

1. ONE-DIMENSIONAL ANALYSIS
2. EFFECTIVE MODULUS and
CONSTANT VISCOUS DAMPING
NOT A FUNCTION OF TIME
3. PERMANENT SET NOT POSSIBLE
4. ARE THESE APPROXIMATIONS
NECESSARY ? Use **SAP 2000**

STRUCTURAL ENGINEER'S VIEW OF SOIL-STRUCTURE SYSTEM



GEOTECHNICAL ENGINEER'S VIEW OF SOIL-STRUCTURE SYSTEM



**WHAT IS THE MOST SIGNIFICANT
BARRIER TO PRODUCING GOOD
SOLUTIONS OF SOIL-STRUCTURE
INTERACTION PROBLEMS?**

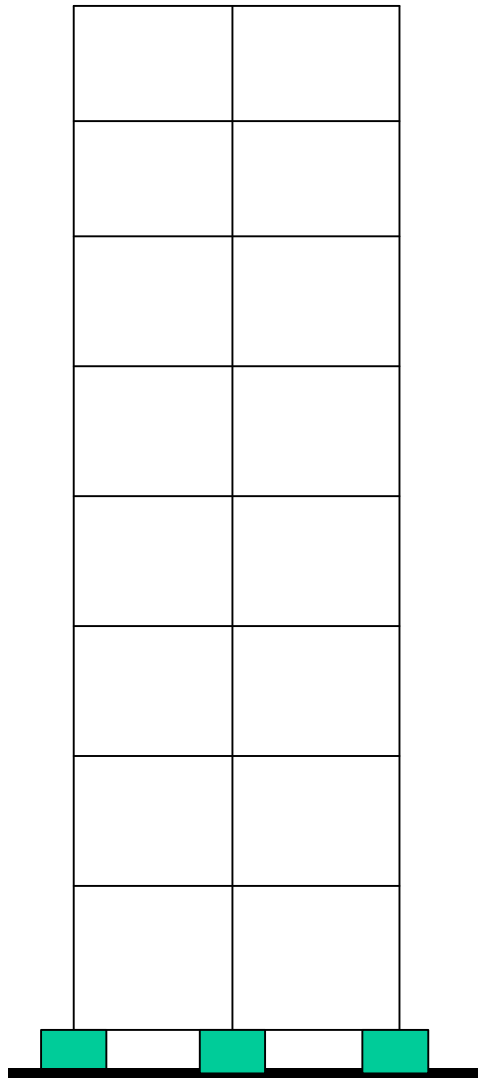
**SITE RESPONSE AND STRUCTURAL
ENGINEERING ARE CONDUCTED AT
DIFFERENT LOCATIONS (OFFICES)
USING DIFFERENT NUMERICAL
METHODS AND APPROXIMATIONS**

WIND RESPONSE

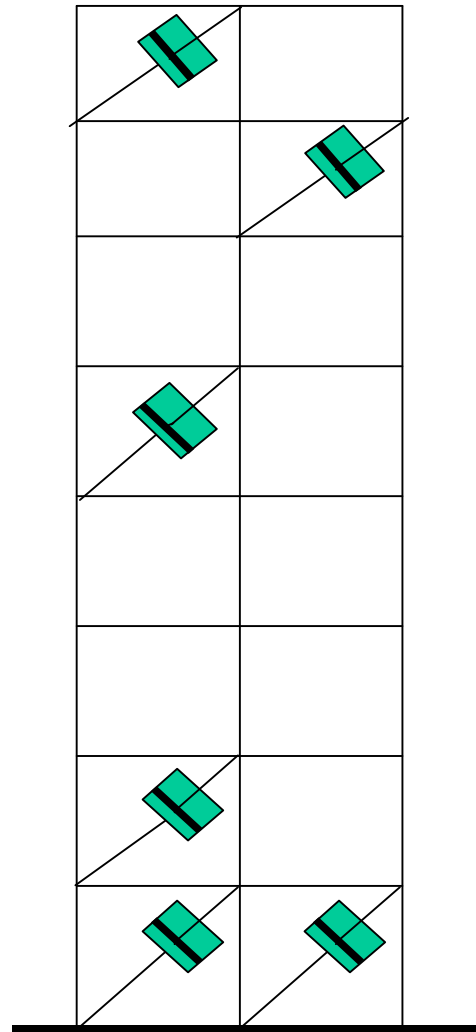
OF

TALL BUILDINGS

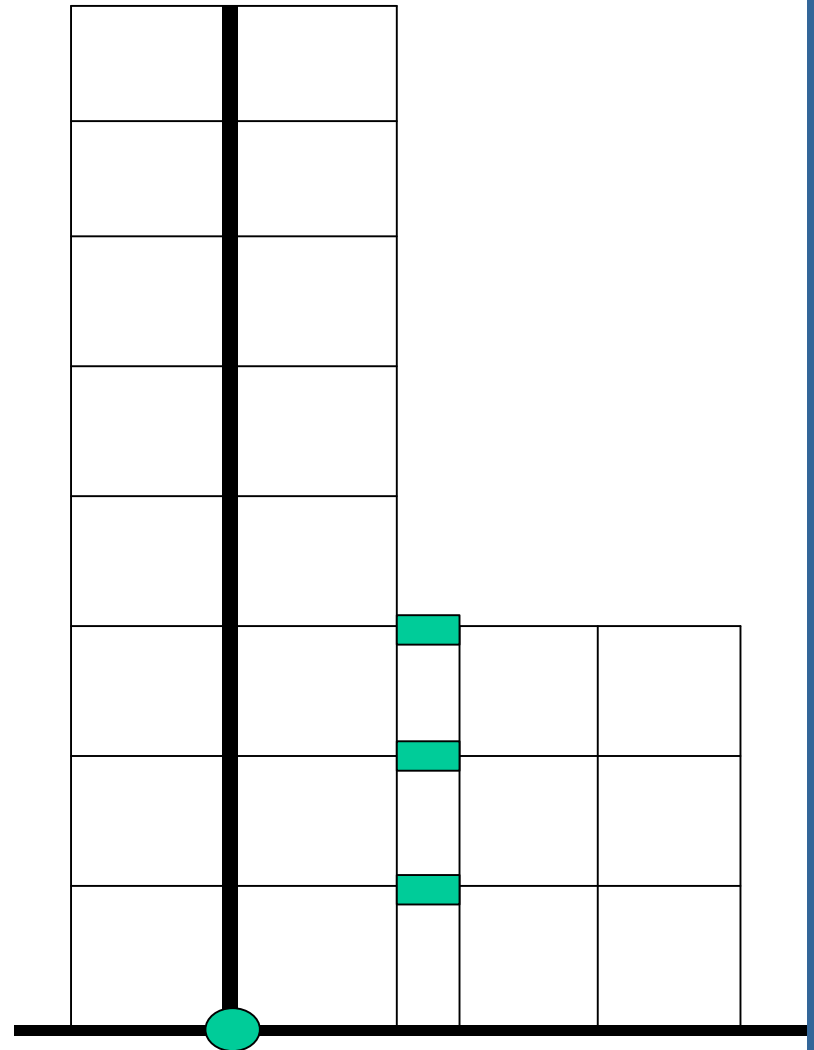
ENERGY DISSIPATION SYSTEMS



Base Isolation
Or Uplift



Dampers



Plastic Hinge - Friction
and Gap Elements

Dynamic Wind Analysis

1. Random Vibration
 - A. Classical Approach.
 - B. Linear Analysis Only
2. Time History Response
 - A. Exact For Given Periodic Loading
 - B. Non-linear Analysis Is Possible
 - C. Can Perform Code Checks As A Function Of Time

Weakness Of The Response Spectrum Methods

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - \frac{f_a}{F_{ex}}) F_{bx}} + \frac{C_{my} f_{by}}{(1 - \frac{f_a}{F_{ey}}) F_{by}} \leq 1.0$$

The Use Of The Maximum Peak Values Of f_a , f_{bx} and f_{by} Produces An Inconsistent Design

Axial Members Are Under Designed Compared To Bi-Axial Bending Members

SOLUTION ?

Use Design Checks As A Function Of Time

Determination Of Wind Forces As Function Of Time

$$\mathbf{R}(t) = \sum \mathbf{f}_i \mathbf{g}(t)_i$$

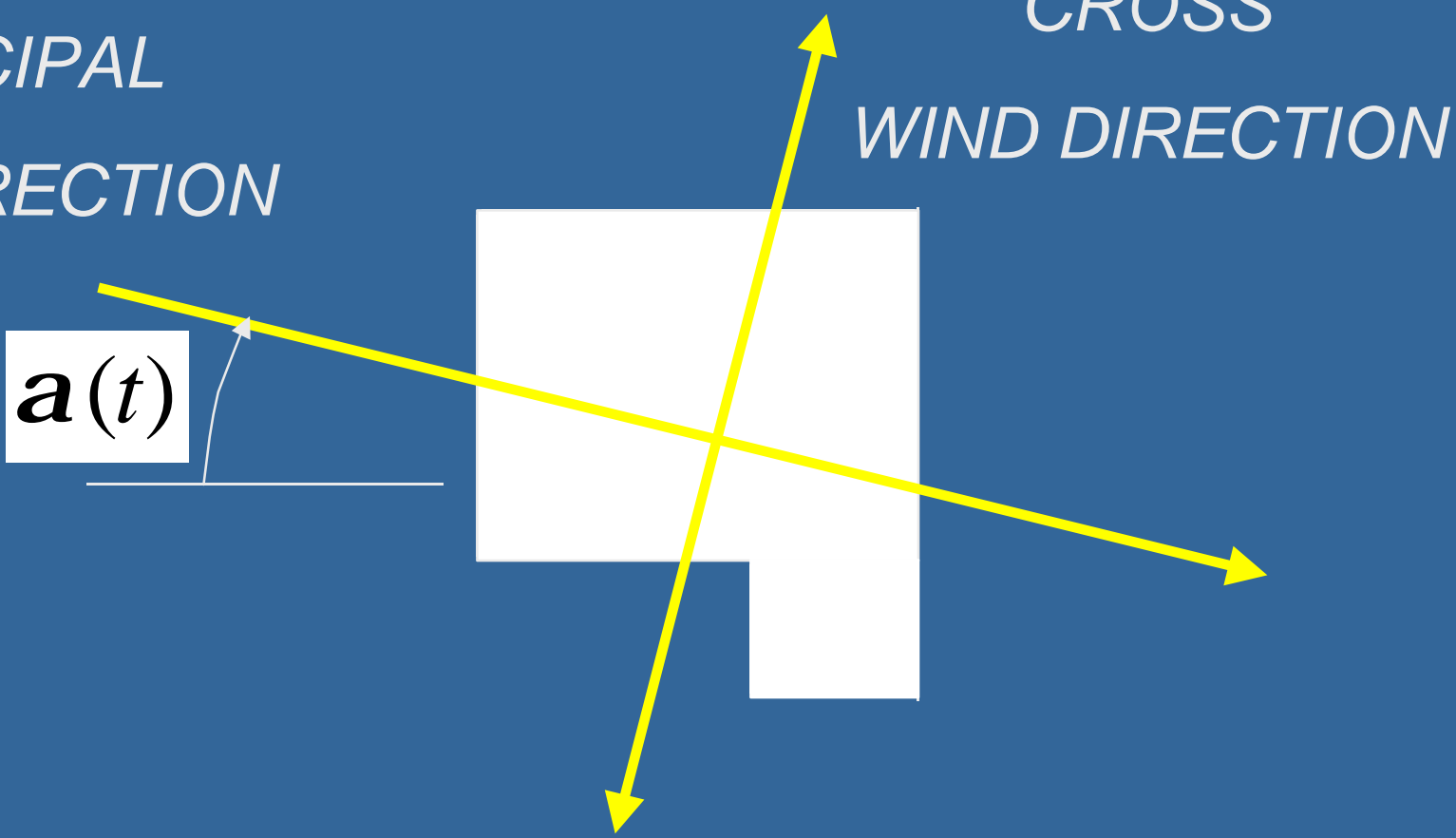
1. ANALYSIS AND FORMULAS
2. FIELD MEASUREMENTS
3. WIND TUNNEL TESTS

WIND FORCES ACTING ON BUILDINGS

*PRINCIPAL
WIND DIRECTION*

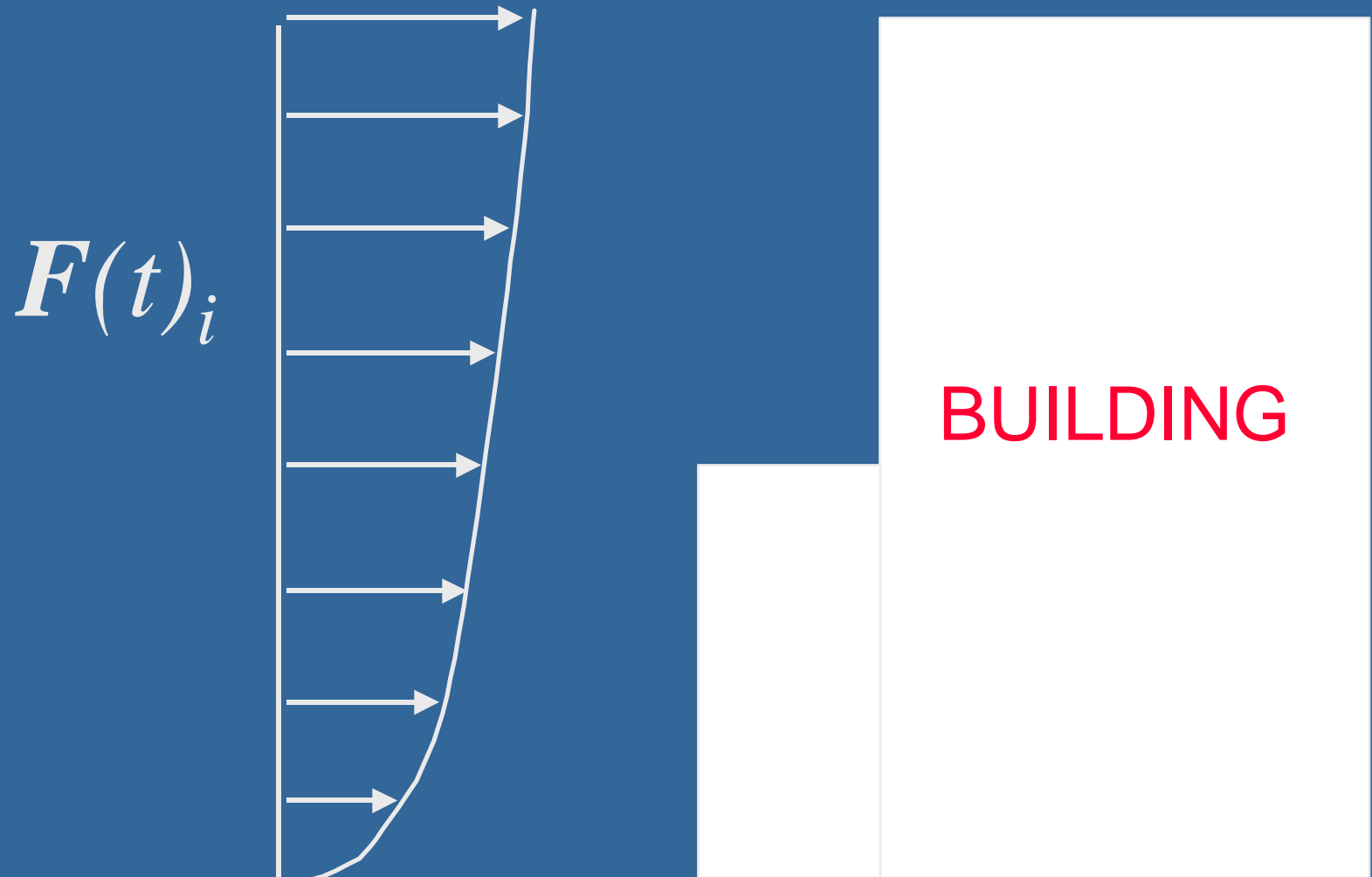
*CROSS
WIND DIRECTION*

$a(t)$

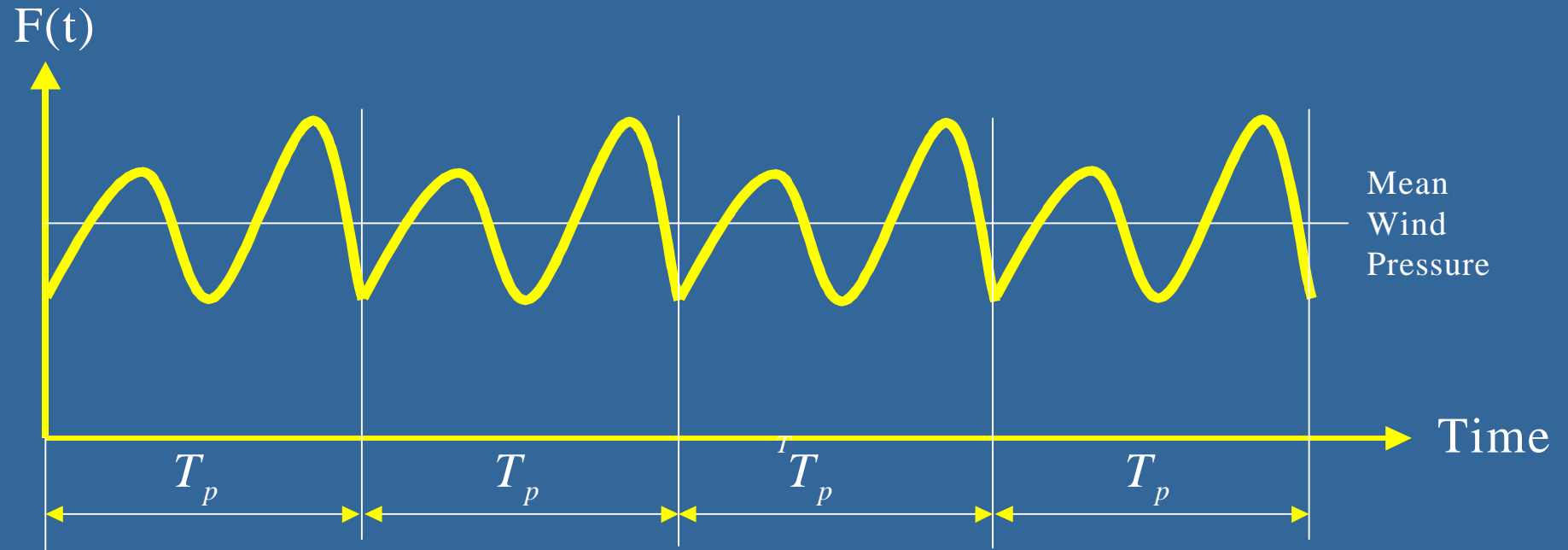


The diagram shows a white building silhouette on a blue background. Two yellow arrows represent wind directions. One arrow, labeled 'PRINCIPAL WIND DIRECTION', points from the upper left towards the building. The other arrow, labeled 'CROSS WIND DIRECTION', points from the upper right towards the building. A white box containing the mathematical expression $a(t)$ is positioned to the left of the principal wind arrow. A horizontal white line is drawn below the $a(t)$ box, and a curved white arrow indicates the angle between this line and the principal wind arrow.

VERTICAL DISTRIBUTION OF WIND FORCES

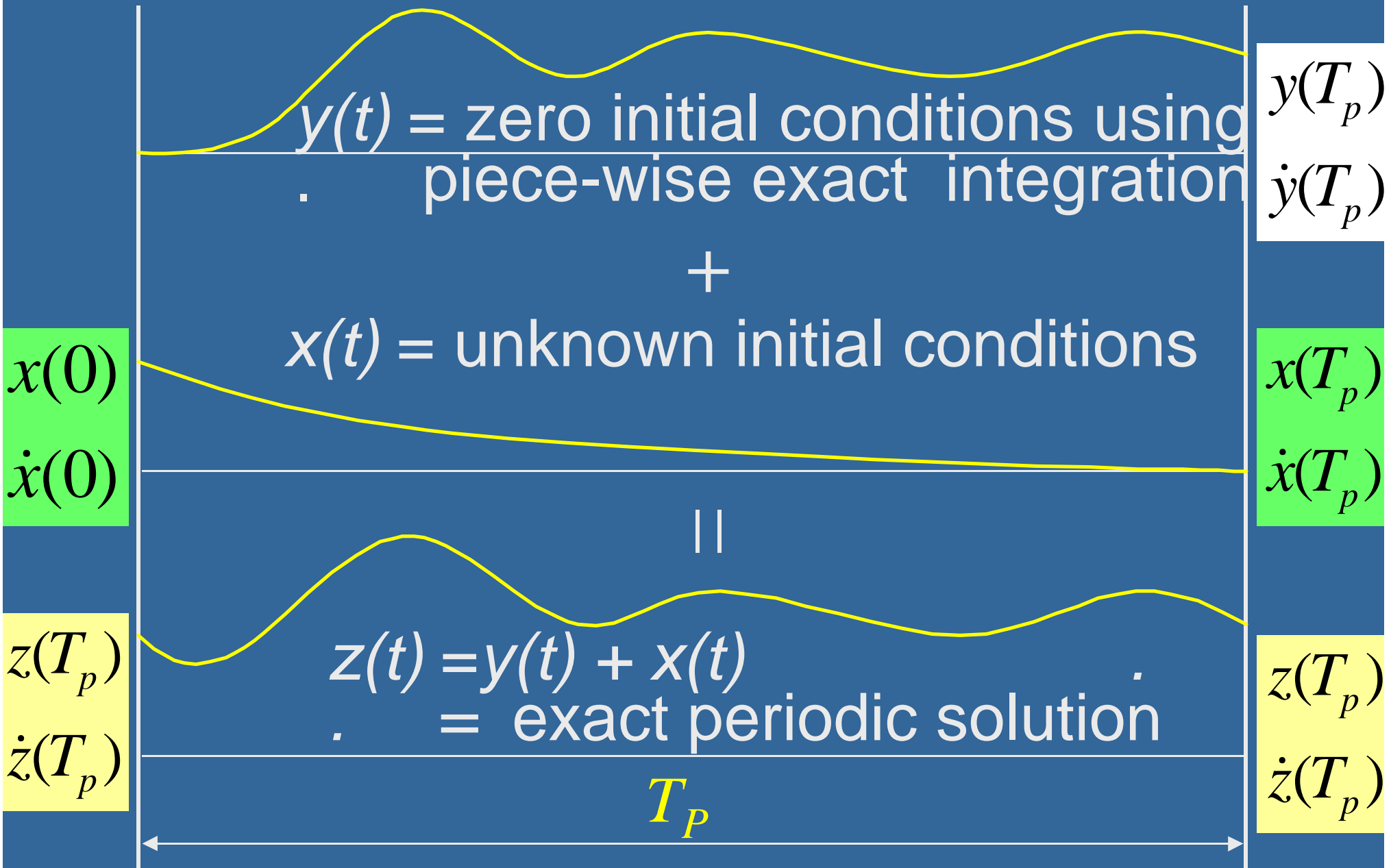


PERIODIC WIND LOADING



$T_p = 10$ TO 50 Seconds

Conversion Of Transient Solution To Periodic Solution

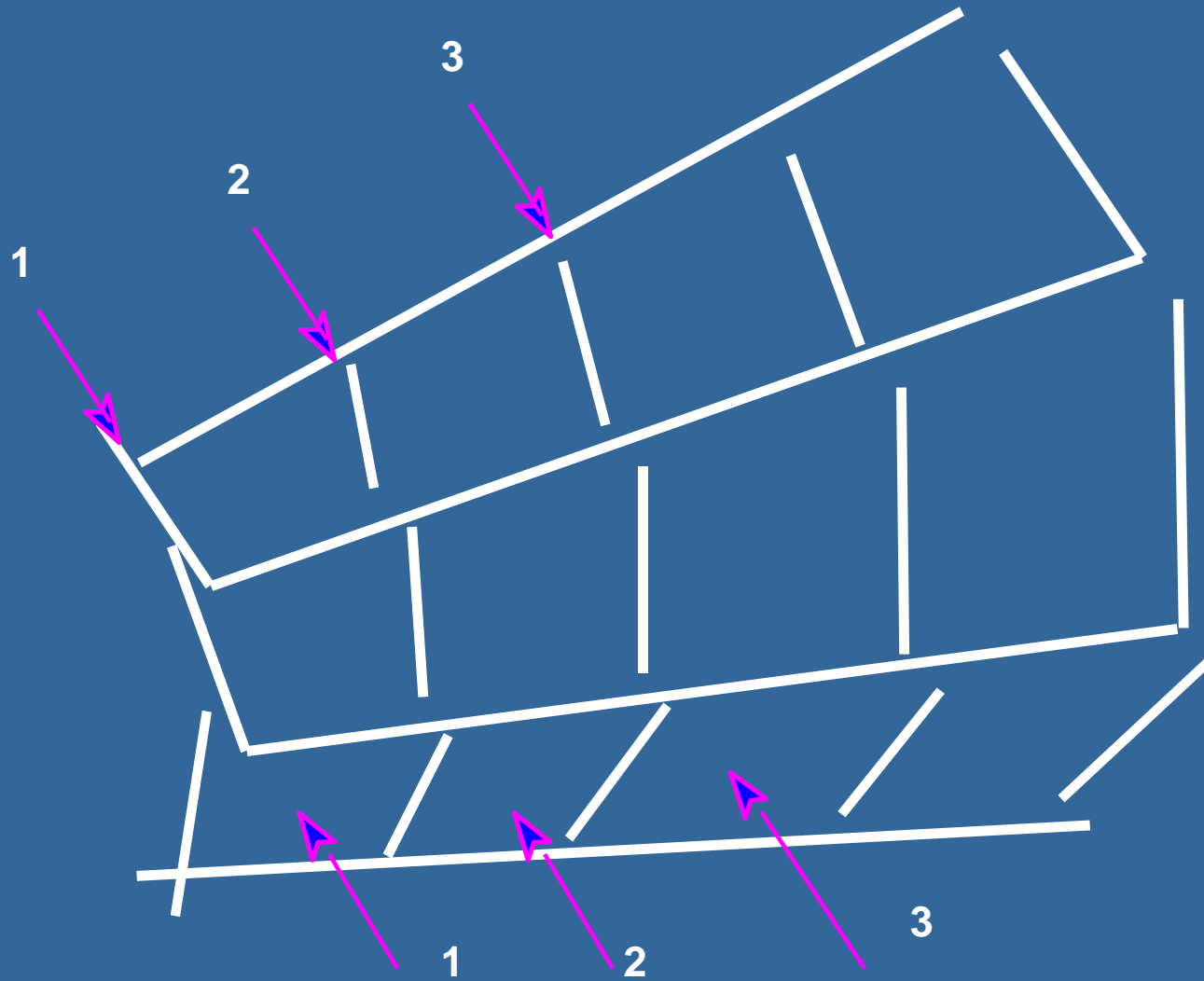


PARALLEL ENGINEERING

AND

PARALLEL COMPUTERS

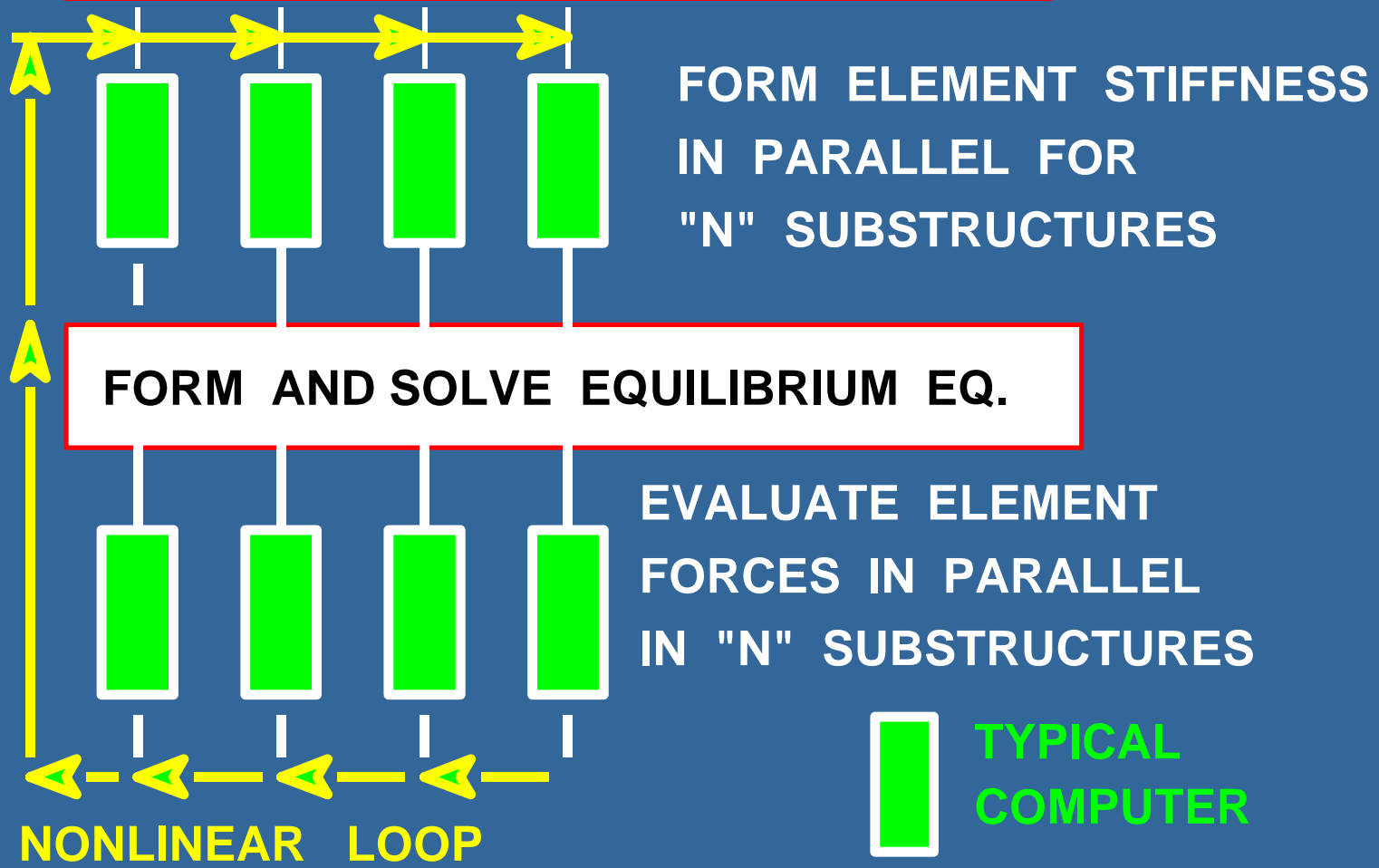
ONE PROCESSOR ASSIGNED TO EACH JOINT



ONE PROCESSOR ASSIGNED TO EACH MEMBER

PARALLEL STRUCTURAL ANALYSIS

DIVIDE STRUCTURE INTO "N" DOMAINS



FINAL REMARKS

1. LINEAR AND NONLINEAR DYNAMIC ANALYSES CAN BE CONDUCTED, OF LARGE STRUCTURES, USING INEXPENSIVE PERSONAL COMPUTERS
2. SUBSTRUCTURE METHODS HAS MANY ADVANTAGES FOR LARGE STRUCTURES
3. TIME-HISTORY DYNAMIC WIND ANALYSES CAN NOW BE CONDUCTED OF STRUCTURES
4. NEW NUMERICAL METHODS ALLOW FOR **F**AST **N**ONLINEAR **A**NALYSIS FOR MANY STRUCTURES SUBJECTED TO EARTHQUAKE LOADING

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